Module 0 Review and Self-Test

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Practice problems

1. Find the angle θ between the vectors $\mathbf{u} = [1, 1, 0]$ and $\mathbf{v} = [0, 1, 1]$.
2. Find the area of the triangle with vertices $A(0, 0, 0)$, $B(2, 0, 0)$, and $C(0, 3, 0)$.
3. A plane has the equation $2x - y + 2z = 4$. Find the shortest distance from the point $P(1, 2, 1)$ to this plane.

Solutions

1. Find the angle θ between the vectors $\mathbf{u} = [1, 1, 0]$ and $\mathbf{v} = [0, 1, 1]$.

   - Compute the dot product: $\mathbf{u} \cdot \mathbf{v} = (1)(0) + (1)(1) + (0)(1) = 1$.
   - Compute the lengths: $|\mathbf{u}| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$ and $|\mathbf{v}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$.
   - Use the formula $\cos(\theta) = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$.
   - Find the angle: $\theta = \arccos(1/2) = 60°$.

   Answer: 60ˆ\circ

2. Find the area of the triangle with vertices $A(0, 0, 0)$, $B(2, 0, 0)$, and $C(0, 3, 0)$.

   - Find vectors $\vec{AB} = [2, 0, 0]$ and $\vec{AC} = [0, 3, 0]$.
   - Compute the cross product: $\vec{AB} \times \vec{AC} = [0, 0, (2)(3) - (0)(0)] = [0, 0, 6]$.
   - The length of the cross product is $\sqrt{0^2 + 0^2 + 6^2} = 6$.
   - The area of the triangle is half the area of the parallelogram: $6/2 = 3$.

   Answer: 3

3. A plane has the equation $2x - y + 2z = 4$. Find the shortest distance from the point $P(1, 2, 1)$ to this plane.

   - Identify the normal vector $\mathbf{n} = [2, -1, 2]$.
   - Find the length of the normal: $|\mathbf{n}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3$.
   - Find a point A on the plane. If $y = 0, z = 0$, then $2x = 4 \implies x = 2$. So $A(2, 0, 0)$.
   - Compute the vector $\vec{AP} = P - A = [1 - 2, 2 - 0, 1 - 0] = [-1, 2, 1]$.
   - The distance is $d = \frac{|\vec{AP}\cdot\mathbf{n}|}{|\mathbf{n}|} = \frac{|(-1)(2)+(2)(-1)+(1)(2)|}{3} = \frac{|-2-2+2|}{3} = \frac{2}{3}$.

   Answer: 2/3
