

From arrows to data: vectors as information

Lesson 13 · Module 0 — Vectors & Geometry · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. A student has a portfolio of two assets. Asset A is worth 10 units and Asset B is worth 50 units. The risk weights for these assets are 0.3 and 0.7 respectively. Calculate the total weighted risk using the dot product of the weight vector $\mathbf{w} = [0.3, 0.7]$ and the portfolio vector $\mathbf{p} = [10, 50]$.
2. Two documents are represented by word-frequency vectors. Document 1 is $\mathbf{d}_1 = [3, 0]$ and Document 2 is $\mathbf{d}_2 = [2, 2]$. Calculate the cosine similarity between these two documents.
3. Three points in a 3D feature space are $A = [1, 2, 1]$, $B = [2, 2, 2]$, and $C = [1, 0, 1]$. Which point is closer to A : B or C ?

Solutions

1. A student has a portfolio of two assets. Asset A is worth 10 units and Asset B is worth 50 units. The risk weights for these assets are 0.3 and 0.7 respectively. Calculate the total weighted risk using the dot product of the weight vector $\mathbf{w} = [0.3, 0.7]$ and the portfolio vector $\mathbf{p} = [10, 50]$.

- Set up the dot product: $0.3 \times 10 + 0.7 \times 50$
- Calculate the first term: $0.3 \times 10 = 3$
- Calculate the second term: $0.7 \times 50 = 35$
- Sum the results: $3 + 35 = 38$

Answer: 38

2. Two documents are represented by word-frequency vectors. Document 1 is $\mathbf{d}_1 = [3, 0]$ and Document 2 is $\mathbf{d}_2 = [2, 2]$. Calculate the cosine similarity between these two documents.

- Find the length of $\mathbf{d}_1$: $|\mathbf{d}_1| = \sqrt{3^2 + 0^2} = 3$
- Find the length of $\mathbf{d}_2$: $|\mathbf{d}_2| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$
- Compute the dot product: $\mathbf{d}_1 \cdot \mathbf{d}_2 = (3 \times 2) + (0 \times 2) = 6$
- Divide by the product of lengths: $6/(3 \times 2\sqrt{2}) = 1/\sqrt{2} \approx 0.707$

Answer: $\frac{1}{\sqrt{2}} \approx 0.707$

3. Three points in a 3D feature space are $A = [1, 2, 1]$, $B = [2, 2, 2]$, and $C = [1, 0, 1]$. Which point is closer to A: B or C?

- Calculate distance AB: $\sqrt{(2-1)^2 + (2-2)^2 + (2-1)^2} = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2} \approx 1.414$
- Calculate distance AC: $\sqrt{(1-1)^2 + (0-2)^2 + (1-1)^2} = \sqrt{0^2 + (-2)^2 + 0^2} = \sqrt{4} = 2$
- Compare the distances: $\sqrt{2} < 2$

Answer: Point B is closer to A