

Solving geometry problems with vectors — clinic

Lesson 12 · Module 0 — Vectors & Geometry · Linear Algebra · dailymaths.uk/linalg

Practice problems

1. Find the distance from the point $P(2, 3, 4)$ to the plane $x + y + z = 1$.
2. Find the centroid of a triangle with vertices $A(1, 2, 0)$, $B(5, 0, 2)$, and $C(3, 6, 4)$.
3. Find the distance from the point $P(1, 1, 1)$ to the line passing through $A(0, 0, 0)$ with direction $\mathbf{v} = [0, 1, 0]$.

Solutions

1. Find the distance from the point $P(2, 3, 4)$ to the plane $x + y + z = 1$.

- Identify the normal vector $\mathbf{n} = [1, 1, 1]$.
- Find a point A on the plane, e.g., $A(1, 0, 0)$.
- Compute the vector $\mathbf{AP} = [2 - 1, 3 - 0, 4 - 0] = [1, 3, 4]$.
- Compute the unit normal $\hat{\mathbf{n}} = \frac{1}{\sqrt{3}}[1, 1, 1]$.
- Calculate the distance $d = |\mathbf{AP} \cdot \hat{\mathbf{n}}| = \frac{|1(1) + 3(1) + 4(1)|}{\sqrt{3}} = \frac{8}{\sqrt{3}}$.

Answer: $\frac{8}{\sqrt{3}}$

2. Find the centroid of a triangle with vertices $A(1, 2, 0)$, $B(5, 0, 2)$, and $C(3, 6, 4)$.

- Sum the x-coordinates: $1 + 5 + 3 = 9$. Divide by 3 to get 3.
- Sum the y-coordinates: $2 + 0 + 6 = 8$. Divide by 3 to get $8/3$.
- Sum the z-coordinates: $0 + 2 + 4 = 6$. Divide by 3 to get 2.

Answer: $(3, 8/3, 2)$

3. Find the distance from the point $P(1, 1, 1)$ to the line passing through $A(0, 0, 0)$ with direction $\mathbf{v} = [0, 1, 0]$.

- Find the vector $\mathbf{AP} = [1, 1, 1]$.
- The unit direction vector is $\hat{\mathbf{v}} = [0, 1, 0]$.
- Compute the cross product $\mathbf{AP} \times \hat{\mathbf{v}} = [1(0) - 1(1), 1(0) - 1(0), 1(1) - 1(0)] = [-1, 0, 1]$.
- Calculate the magnitude $d = \sqrt{(-1)^2 + 0^2 + 1^2} = \sqrt{2}$.

Answer: $\sqrt{2}$