Planes in 3D: normal vectors and equations

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Practice problems

1. Find the equation of the plane with normal vector $\mathbf{n} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ that passes through the point $(4, 1, 5)$.
2. Determine if the vector $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is parallel to the plane $3x - y + 2z = 14$.
3. Find the equation of the plane passing through the points $P(2, 0, 0)$, $Q(0, 3, 0)$, and $R(0, 0, 6)$.

Solutions

1. Find the equation of the plane with normal vector $\mathbf{n} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ that passes through the point $(4, 1, 5)$.

   - Use the point-normal form: $2(x-4) - 3(y-1) + 1(z-5) = 0$.
   - Expand the terms: $2x - 8 - 3y + 3 + z - 5 = 0$.
   - Combine constants: $2x - 3y + z - 10 = 0$.
   - Move the constant to the right side: $2x - 3y + z = 10$.

   Answer: 2x - 3y + z = 10

2. Determine if the vector $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is parallel to the plane $3x - y + 2z = 14$.

   - Identify the normal vector of the plane: $\mathbf{n} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$.
   - A vector is parallel to the plane if it is perpendicular to the normal vector.
   - Compute the dot product: $\mathbf{n} \cdot \mathbf{v} = (3)(1) + (-1)(2) + (2)(1)$.
   - Calculate the sum: $3 - 2 + 2 = 3$.
   - Since the dot product is not zero, the vector is not parallel to the plane.

   Answer: No, it is not parallel.

3. Find the equation of the plane passing through the points $P(2, 0, 0)$, $Q(0, 3, 0)$, and $R(0, 0, 6)$.

   - Find two vectors in the plane: $\mathbf{PQ} = Q - P = \begin{pmatrix} -2 \\ 3 \\ 0 \end{pmatrix}$ and $\mathbf{PR} = R - P = \begin{pmatrix} -2 \\ 0 \\ 6 \end{pmatrix}$.
   - Find the normal vector using the cross product: $\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} = \begin{pmatrix} (3)(6) - (0)(0) \\ (0)(-2) - (-2)(6) \\ (-2)(0) - (3)(-2) \end{pmatrix} = \begin{pmatrix} 18 \\ 12 \\ 6 \end{pmatrix}$.
   - Simplify the normal vector by dividing by 6: $\mathbf{n}_{simple} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$.
   - Use point $P(2, 0, 0)$ in the equation: $3(x-2) + 2(y-0) + 1(z-0) = 0$.
   - Simplify: $3x - 6 + 2y + z = 0 \implies 3x + 2y + z = 6$.

   Answer: 3x + 2y + z = 6
