Lines in 2D and 3D: parametric and vector forms

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Practice problems

1. Find the vector equation of the line in 2D that passes through the point $(-2, 5)$ and has a direction vector $\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.
2. Find the parametric equations of the line in 3D passing through the points $A(1, 0, 2)$ and $B(3, -2, 5)$.
3. Convert the Cartesian equation $y = -3x + 4$ into a vector equation $\mathbf{r}(t) = \mathbf{p} + t\mathbf{v}$.

Solutions

1. Find the vector equation of the line in 2D that passes through the point $(-2, 5)$ and has a direction vector $\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

   - Identify the starting point $\mathbf{p} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$.
   - Identify the direction vector $\mathbf{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.
   - Substitute into the formula $\mathbf{r}(t) = \mathbf{p} + t\mathbf{v}$.

   Answer: \mathbfr(t) = \beginpmatrix -2 \\ 5 \endpmatrix + t \beginpmatrix 4 \\ 3 \endpmatrix

2. Find the parametric equations of the line in 3D passing through the points $A(1, 0, 2)$ and $B(3, -2, 5)$.

   - Calculate the direction vector $\mathbf{v} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3-1 \\ -2-0 \\ 5-2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$.
   - Use point A as the starting point $\mathbf{p} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$.
   - Write the components: $x = 1 + 2t$, $y = 0 - 2t$, $z = 2 + 3t$.

   Answer: x = 1 + 2t, y = -2t, z = 2 + 3t

3. Convert the Cartesian equation $y = -3x + 4$ into a vector equation $\mathbf{r}(t) = \mathbf{p} + t\mathbf{v}$.

   - Find a point on the line: if $x = 0$, then $y = 4$, so $\mathbf{p} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$.
   - Identify the slope $m = -3$, which means for $\Delta x = 1$, $\Delta y = -3$.
   - The direction vector is $\mathbf{v} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.
   - Combine into the vector form.

   Answer: \mathbfr(t) = \beginpmatrix 0 \\ 4 \endpmatrix + t \beginpmatrix 1 \\ -3 \endpmatrix

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