The cross product (3D) — definition and geometry

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Practice problems

1. Compute the cross product of $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$.

2. Find the area of the parallelogram spanned by $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$.

3. Determine if the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$ are parallel using the cross product.

Solutions

1. Compute the cross product of $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$.

    - The first component is $(2)(3) - (0)(1) = 6$.
    - The second component is $(0)(0) - (1)(3) = -3$.
    - The third component is $(1)(1) - (2)(0) = 1$.

    Answer: \beginpmatrix 6 \\ -3 \\ 1 \endpmatrix

2. Find the area of the parallelogram spanned by $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$.

    - Compute the cross product: $\mathbf{u} \times \mathbf{v} = \begin{pmatrix} (0)(0) - (0)(3) \\ (0)(0) - (2)(0) \\ (2)(3) - (0)(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$.
    - The area is the magnitude of this vector: $\sqrt{0^2 + 0^2 + 6^2} = 6$.

    Answer: 6

3. Determine if the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$ are parallel using the cross product.

    - Compute the first component: $(-1)(-4) - (2)(2) = 4 - 4 = 0$.
    - Compute the second component: $(2)(-2) - (1)(-4) = -4 + 4 = 0$.
    - Compute the third component: $(1)(2) - (-1)(-2) = 2 - 2 = 0$.
    - Since the cross product is the zero vector, the vectors are parallel.

    Answer: Yes, they are parallel
