

The dot product II — angles, projection, and the geometric meaning

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Practice problems

1. Determine if the vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$ are perpendicular.
2. Find the angle θ between $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
3. Given two unit vectors $\mathbf{u}$ and $\mathbf{v}$, their dot product is -0.5 . What is the angle between them?

Solutions

1. Determine if the vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$ are perpendicular.

- Compute the dot product: $\mathbf{u} \cdot \mathbf{v} = (2)(-6) + (3)(4)$
- Calculate the values: $-12 + 12 = 0$
- Since the dot product is 0, the vectors are perpendicular.

Answer: Yes, they are perpendicular.

2. Find the angle θ between $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

- Compute the dot product: $\mathbf{u} \cdot \mathbf{v} = (1)(1) + (0)(1) = 1$
- Compute magnitudes: $|\mathbf{u}| = \sqrt{1^2 + 0^2} = 1$ and $|\mathbf{v}| = \sqrt{1^2 + 1^2} = \sqrt{2}$
- Use the formula: $\cos(\theta) = \frac{1}{1 \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$
- Find the angle: $\theta = \arccos(\frac{1}{\sqrt{2}}) = 45^\circ$

Answer: 45°

3. Given two unit vectors $\mathbf{u}$ and $\mathbf{v}$, their dot product is -0.5 . What is the angle between them?

- For unit vectors, $\mathbf{u} \cdot \mathbf{v} = \cos(\theta)$
- Set $\cos(\theta) = -0.5$
- Find the angle: $\theta = \arccos(-0.5) = 120^\circ$

Answer: 120°