The dot product I — definition and computation

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Practice problems

1. Compute the dot product of $\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$.
2. Determine if the vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$ are orthogonal.
3. Compute the dot product of the 3D vectors $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$.

Solutions

1. Compute the dot product of $\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

   - Multiply the first components: $5 \times (-3) = -15$.
   - Multiply the second components: $2 \times 4 = 8$.
   - Sum the results: $-15 + 8 = -7$.

   Answer: -7

2. Determine if the vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$ are orthogonal.

   - Compute the dot product: $(2)(-6) + (3)(4)$.
   - Calculate the products: $-12 + 12$.
   - The sum is 0. Since the dot product is zero, the vectors are orthogonal.

   Answer: Yes, they are orthogonal

3. Compute the dot product of the 3D vectors $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$.

   - Multiply first components: $1 \times 3 = 3$.
   - Multiply second components: $-1 \times 4 = -4$.
   - Multiply third components: $2 \times 0 = 0$.
   - Sum the results: $3 + (-4) + 0 = -1$.

   Answer: -1
