Span: what can you reach?

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Practice problems

1. Determine if the vector $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ is in the span of $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. If so, find the weights c_1 and c_2.
2. Describe the span of the vectors $\mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$. Is it a line or a plane?
3. Find a vector in $\mathbb{R}^2$ that is NOT in the span of $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. Explain why.

Solutions

1. Determine if the vector $\begin{bmatrix}5\\1\end{bmatrix}$ is in the span of $\mathbf{v} = \begin{bmatrix}1\\0\end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix}1\\1\end{bmatrix}$. If so, find the weights c_1 and c_2.

   - Set up the equation $c_1 \begin{bmatrix}1\\0\end{bmatrix} + c_2 \begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix}$.
   - From the second component, $0 \cdot c_1 + 1 \cdot c_2 = 1$, so $c_2 = 1$.
   - Substitute $c_2 = 1$ into the first component: $1 \cdot c_1 + 1 \cdot 1 = 5$.
   - Solve for c_1: $c_1 = 5 - 1 = 4$.
   - Since we found weights $c_1 = 4$ and $c_2 = 1$, the vector is in the span.

   Answer: Yes, $c_1 = 4, c_2 = 1$

2. Describe the span of the vectors $\mathbf{v} = \begin{bmatrix}3\\-2\end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix}-6\\4\end{bmatrix}$. Is it a line or a plane?

   - Check if $\mathbf{w}$ is a scalar multiple of $\mathbf{v}$.
   - Observe that $-2 \cdot \begin{bmatrix}3\\-2\end{bmatrix} = \begin{bmatrix}-6\\4\end{bmatrix}$.
   - Since $\mathbf{w} = -2\mathbf{v}$, the vectors are collinear.
   - The span of collinear vectors is a line passing through the origin.

   Answer: A line

3. Find a vector in $\mathbb{R}^2$ that is NOT in the span of $\mathbf{v} = \begin{bmatrix}1\\2\end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix}2\\4\end{bmatrix}$. Explain why.

   - Notice that $\mathbf{w} = 2\mathbf{v}$, so the span is the line $y = 2x$.
   - Any vector where the second component is not twice the first component will be outside the span.
   - For example, choose $\begin{bmatrix}1\\1\end{bmatrix}$.
   - Check: $c_1 \begin{bmatrix}1\\2\end{bmatrix} + c_2 \begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}c_1 + 2c_2\\2c_1 + 4c_2\end{bmatrix}$.
   - The second component is always twice the first. Since $1 \neq 2(1)$, the vector $\begin{bmatrix}1\\1\end{bmatrix}$ is not in the span.

   Answer: Any vector where $y \neq 2x$, e.g., $\begin{bmatrix}1\\1\end{bmatrix}$
