Linear combinations: the most important idea in the course

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Practice problems

1. Compute the linear combination $3\mathbf{v} + 4\mathbf{w}$ where $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.
2. Find the resulting vector for the linear combination $-2\mathbf{v} + 3\mathbf{w}$ given $\mathbf{v} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.
3. If $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, what weights c_1 and c_2 are needed to reach the point $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$?

Solutions

1. Compute the linear combination $3\mathbf{v} + 4\mathbf{w}$ where $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

    - Scale $\mathbf{v}$ by 3: $3 \cdot \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$.
    - Scale $\mathbf{w}$ by 4: $4 \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$.
    - Add the results: $\begin{bmatrix} 6 \\ -3 \end{bmatrix} + \begin{bmatrix} 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 10 \\ 9 \end{bmatrix}$.

    Answer: \beginbmatrix 10 \\ 9 \endbmatrix

2. Find the resulting vector for the linear combination $-2\mathbf{v} + 3\mathbf{w}$ given $\mathbf{v} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

    - Scale $\mathbf{v}$ by -2: $-2 \cdot \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$.
    - Scale $\mathbf{w}$ by 3: $3 \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$.
    - Add the results: $\begin{bmatrix} 2 \\ -8 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ -8 \end{bmatrix}$.

    Answer: \beginbmatrix 8 \\ -8 \endbmatrix

3. If $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, what weights c_1 and c_2 are needed to reach the point $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$?

    - Set up the equation: $c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.
    - From the first component: $c_1(1) + c_2(0) = 3$, so $c_1 = 3$.
    - From the second component: $c_1(1) + c_2(1) = 5$.
    - Substitute $c_1 = 3$ into the second equation: $3 + c_2 = 5$, so $c_2 = 2$.

    Answer: c_1 = 3, c_2 = 2
