

What linear algebra is: the mathematics of flat things

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Practice problems

1. A transformation is linear if it keeps the origin fixed and keeps parallel lines parallel. If we slide the entire grid two units to the right, is this a linear transformation?
2. Imagine a transformation that stretches the x-axis by a factor of two but leaves the y-axis alone. Does this keep parallel lines parallel?
3. If a linear transformation collapses a 2D grid onto a single line passing through the origin, what happens to the area of the shapes on that grid?

Solutions

1. A transformation is linear if it keeps the origin fixed and keeps parallel lines parallel. If we slide the entire grid two units to the right, is this a linear transformation?

- Check the origin: the point $(0, 0)$ moves to $(2, 0)$.
- Since the origin does not stay fixed, the rule is violated.

Answer: No

2. Imagine a transformation that stretches the x-axis by a factor of two but leaves the y-axis alone. Does this keep parallel lines parallel?

- Any two parallel lines have the same slope.
- Stretching one axis uniformly changes the slope of all lines by the same ratio.
- Therefore, lines that were parallel remain parallel.

Answer: Yes

3. If a linear transformation collapses a 2D grid onto a single line passing through the origin, what happens to the area of the shapes on that grid?

- A line has zero width.
- Any 2D shape flattened into a 1D line loses its area.
- The resulting area is zero.

Answer: The area becomes zero