

Exponential growth and decay models

Lesson 95 · Module 4 — Applications of Integration · The Calculus Course · dailymaths.uk/course

Practice problems

1. A radioactive isotope has a half-life of 1600 years. If a sample initially contains 50 grams, how much remains after 2000 years?
2. A population of fish in a pond grows exponentially. Initially, there are 200 fish. After 2 years, there are 350 fish. Find the doubling time of the population.
3. A metal rod at 150°C is placed in a room at 20°C . After 5 minutes, the rod is 100°C . How long will it take for the rod to reach 40°C ?

Solutions

1. A radioactive isotope has a half-life of 1600 years. If a sample initially contains 50 grams, how much remains after 2000 years?

- Find the decay constant k using $k = \frac{-\ln 2}{1600}$.
- Set up the model $y(t) = 50e^{kt}$.
- Substitute $t = 2000$ into the equation: $y(2000) = 50e^{\frac{-\ln 2}{1600} \cdot 2000}$.
- Simplify the exponent: $y(2000) = 50e^{-1.25 \ln 2} = 50(2^{-1.25})$.
- Calculate the final value: $y(2000) \approx 21.02$ grams.

Answer: 21.02 \text{ g}

2. A population of fish in a pond grows exponentially. Initially, there are 200 fish. After 2 years, there are 350 fish. Find the doubling time of the population.

- Use the model $y(t) = 200e^{kt}$ and the condition $y(2) = 350$.
- Solve for k : $350 = 200e^{2k} \implies 1.75 = e^{2k} \implies k = \frac{\ln(1.75)}{2} \approx 0.2798$.
- Use the doubling time formula $t_{\text{double}} = \frac{\ln 2}{k}$.
- Substitute k : $t_{\text{double}} = \frac{\ln 2}{0.2798} \approx 2.48$ years.

Answer: 2.48 \text{ years}

3. A metal rod at 150°C is placed in a room at 20°C . After 5 minutes, the rod is 100°C . How long will it take for the rod to reach 40°C ?

- Use Newton's Law of Cooling: $T(t) = 20 + (150 - 20)e^{kt} = 20 + 130e^{kt}$.
- Find k using $T(5) = 100$: $100 = 20 + 130e^{5k} \implies 80 = 130e^{5k} \implies e^{5k} = \frac{8}{13}$.
- Solve for k : $k = \frac{\ln(8/13)}{5} \approx -0.0937$.
- Set $T(t) = 40$ and solve for t : $40 = 20 + 130e^{-0.0937t} \implies 20 = 130e^{-0.0937t}$.
- Divide by 130: $\frac{2}{13} = e^{-0.0937t} \implies \ln(2/13) = -0.0937t$.
- Calculate t : $t = \frac{\ln(2/13)}{-0.0937} \approx 19.4$ minutes.

Answer: 19.4 \text{ minutes}