

Introduction to differential equations; separable ODEs

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Practice problems

1. Find the general solution for the differential equation $\frac{dy}{dx} = xy$.
2. Solve the initial value problem $\frac{dy}{dx} = 3y$ with the condition $y(0) = 5$.
3. Find the general solution for $\frac{dy}{dx} = \frac{x^2}{y^2}$.

Solutions

1. Find the general solution for the differential equation $\frac{dy}{dx} = xy$.

- Separate the variables by dividing by y and multiplying by dx : $\frac{1}{y} dy = x dx$.
- Integrate both sides: $\int \frac{1}{y} dy = \int x dx$.
- Evaluate the integrals: $\ln |y| = \frac{1}{2}x^2 + C$.
- Exponentiate both sides: $|y| = e^{\frac{1}{2}x^2 + C} = e^C e^{\frac{1}{2}x^2}$.
- Replace e^C with a constant A : $y = Ae^{\frac{1}{2}x^2}$.

Answer: $y = Ae^{\frac{1}{2}x^2}$

2. Solve the initial value problem $\frac{dy}{dx} = 3y$ with the condition $y(0) = 5$.

- Separate the variables: $\frac{1}{y} dy = 3 dx$.
- Integrate both sides: $\ln |y| = 3x + C$.
- Solve for y : $y = Ae^{3x}$.
- Apply the initial condition $y(0) = 5$: $5 = Ae^{3(0)} \implies 5 = A(1) \implies A = 5$.
- Substitute A back into the general solution: $y = 5e^{3x}$.

Answer: $y = 5e^{3x}$

3. Find the general solution for $\frac{dy}{dx} = \frac{x^2}{y^2}$.

- Separate the variables: $y^2 dy = x^2 dx$.
- Integrate both sides: $\int y^2 dy = \int x^2 dx$.
- Evaluate the integrals: $\frac{1}{3}y^3 = \frac{1}{3}x^3 + C$.
- Multiply by 3 to simplify: $y^3 = x^3 + 3C$.
- Let $K = 3C$ and solve for y : $y = \sqrt[3]{x^3 + K}$.

Answer: $y = \sqrt[3]{x^3 + K}$