

Work, force, and fluid pressure

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Practice problems

1. A spring has a spring constant of $k = 500$ Newtons per meter. Find the work done in stretching the spring from $x = 0.1$ meters to $x = 0.3$ meters.
2. A cylindrical tank of radius 1 meter and height 4 meters is full of water (density $\rho = 1000 \text{ kg/m}^3$, $g = 9.8 \text{ m/s}^2$). Find the work required to pump all the water out of the top.
3. A vertical rectangular plate 2 meters wide and 3 meters high is submerged in water. The top of the plate is 1 meter below the surface. Find the total fluid force on the plate (use $\rho g = 9800 \text{ N/m}^3$).

Solutions

1. A spring has a spring constant of $k = 500$ Newtons per meter. Find the work done in stretching the spring from $x = 0.1$ meters to $x = 0.3$ meters.

- The force function is $F(x) = 500x$.
- The work is the integral of $F(x)$ from 0.1 to 0.3: $W = \int_{0.1}^{0.3} 500x \, dx$.
- The antiderivative is $250x^2$.
- Evaluate at the boundaries: $250(0.3)^2 - 250(0.1)^2 = 250(0.09) - 250(0.01)$.
- Calculate the final value: $22.5 - 2.5 = 20$.

Answer: 20 \text{ Joules}

2. A cylindrical tank of radius 1 meter and height 4 meters is full of water (density $\rho = 1000$ kg/m³, $g = 9.8$ m/s²). Find the work required to pump all the water out of the top.

- The volume of a slice at height x is $V = \pi(1)^2 dx = \pi dx$.
- The weight of the slice is $F = 1000 \cdot 9.8 \cdot \pi dx = 9800\pi dx$.
- The distance the slice is lifted is $4 - x$.
- The work integral is $W = \int_0^4 9800\pi(4 - x) \, dx$.
- Integrate: $9800\pi[4x - \frac{1}{2}x^2]_0^4 = 9800\pi(16 - 8) = 9800\pi(8)$.
- Final result: 78400π .

Answer: 78400π \text{ Joules}

3. A vertical rectangular plate 2 meters wide and 3 meters high is submerged in water. The top of the plate is 1 meter below the surface. Find the total fluid force on the plate (use $\rho g = 9800$ N/m³).

- Let h be the depth from the surface. The plate extends from $h = 1$ to $h = 4$.
- The pressure at depth h is $P = 9800h$.
- The area of a thin strip is $w \cdot dh = 2 \, dh$.
- The force integral is $F = \int_1^4 (9800h)(2) \, dh = 19600 \int_1^4 h \, dh$.
- Integrate: $19600[\frac{1}{2}h^2]_1^4 = 9800(4^2 - 1^2)$.
- Calculate: $9800(16 - 1) = 9800(15) = 147000$.

Answer: 147000 \text{ Newtons}