Surface Area of Revolution

Practice problems

1. Find the surface area of the surface generated by rotating the curve $y = x^2$ from $x = 0$ to $x = 1$ around the y-axis.

2. Find the surface area of the surface generated by rotating the line $y = 2x$ from $x = 0$ to $x = 3$ around the x-axis.

3. A curve is defined by $x = \sqrt{y}$ from $y = 0$ to $y = 4$. Find the surface area when this curve is rotated around the y-axis.

Solutions

1. Find the surface area of the surface generated by rotating the curve $y = x^2$ from $x = 0$ to $x = 1$ around the y-axis.

   - Identify the radius for rotation around the y-axis: $r = x$.
   - Find the derivative: $y' = 2x$.
   - Set up the arc length element: $ds = \sqrt{1 + (2x)^2}dx = \sqrt{1 + 4x^2}dx$.
   - Set up the integral: $S = \int_0^1 2\pi x\sqrt{1 + 4x^2}\, dx$.
   - Use u-substitution: $u = 1 + 4x^2$, $du = 8xdx$, so $xdx = \frac{1}{8}du$.
   - Integrate: $S = \frac{2\pi}{8}\int_1^5 u^{1/2}du = \frac{\pi}{4}[\frac{2}{3}u^{3/2}]_1^5 = \frac{\pi}{6}(5\sqrt{5} - 1)$.

   Answer: \frac\pi6(5\sqrt5-1)

2. Find the surface area of the surface generated by rotating the line $y = 2x$ from $x = 0$ to $x = 3$ around the x-axis.

   - Find the derivative: $y' = 2$.
   - Calculate the arc length element: $\sqrt{1 + 2^2} = \sqrt{5}$.
   - Set up the integral: $S = \int_0^3 2\pi(2x)\sqrt{5}\, dx$.
   - Integrate: $S = 4\pi\sqrt{5}\int_0^3 x\, dx = 4\pi\sqrt{5}[\frac{1}{2}x^2]_0^3$.
   - Evaluate: $4\pi\sqrt{5} \cdot \frac{9}{2} = 18\pi\sqrt{5}$.

   Answer: 18\pi\sqrt5

3. A curve is defined by $x = \sqrt{y}$ from $y = 0$ to $y = 4$. Find the surface area when this curve is rotated around the y-axis.

   - The radius is $x = \sqrt{y}$.
   - Find the derivative of x with respect to y: $x' = \frac{1}{2\sqrt{y}}$.
   - Set up the integral: $S = \int_0^4 2\pi\sqrt{y}\sqrt{1 + \frac{1}{4y}}\, dy$.
   - Simplify the integrand: $2\pi\sqrt{y}\frac{\sqrt{4y+1}}{2\sqrt{y}} = \pi\sqrt{4y + 1}$.
   - Integrate: $S = \pi\int_0^4 (4y + 1)^{1/2}\, dy = \pi[\frac{1}{4} \cdot \frac{2}{3}(4y + 1)^{3/2}]_0^4$.
   - Evaluate: $\frac{\pi}{6}(17^{3/2} - 1^{3/2}) = \frac{\pi}{6}(17\sqrt{17} - 1)$.

   Answer: \frac\pi6(17\sqrt17-1)