Arc Length

Practice problems

1. Find the arc length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $[1, 2]$.
2. Find the length of the curve $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 3$.
3. A curve is defined by $y = \ln(\cos x)$ on the interval $[0, \pi/4]$. Find its arc length.

Solutions

1. Find the arc length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ on the interval $[1, 2]$.

   - Compute the derivative: $y' = \frac{3x^2}{6} - \frac{1}{2x^2} = \frac{x^2}{2} - \frac{1}{2x^2}$.
   - Compute $1 + (y')^2$: $1 + (\frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4}) = \frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4}$.
   - Recognize the perfect square: $1 + (y')^2 = (\frac{x^2}{2} + \frac{1}{2x^2})^2$.
   - Integrate: $L = \int_1^2 (\frac{x^2}{2} + \frac{1}{2x^2})\,dx = [\frac{x^3}{6} - \frac{1}{2x}]_1^2$.
   - Evaluate: $(\frac{8}{6} - \frac{1}{4}) - (\frac{1}{6} - \frac{1}{2}) = \frac{7}{6} + \frac{1}{4} = \frac{14+3}{12} = \frac{17}{12}$.

   Answer: 17/12

2. Find the length of the curve $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 3$.

   - Compute the derivative: $y' = \frac{2}{3} \cdot \frac{3}{2}x^{1/2} = x^{1/2}$.
   - Set up the integral: $L = \int_0^3 \sqrt{1 + (x^{1/2})^2}\,dx = \int_0^3 \sqrt{1 + x}\,dx$.
   - Use u-substitution where $u = 1 + x$: $L = [\frac{2}{3}(1 + x)^{3/2}]_0^3$.
   - Evaluate: $\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2} = \frac{2}{3}(8) - \frac{2}{3}(1) = \frac{14}{3}$.

   Answer: 14/3

3. A curve is defined by $y = \ln(\cos x)$ on the interval $[0, \pi/4]$. Find its arc length.

   - Compute the derivative: $y' = \frac{1}{\cos x} \cdot (-\sin x) = -\tan x$.
   - Set up the integral: $L = \int_0^{\pi/4} \sqrt{1 + (-\tan x)^2}\,dx$.
   - Use the identity $1 + \tan^2 x = \sec^2 x$: $L = \int_0^{\pi/4} \sqrt{\sec^2 x}\,dx = \int_0^{\pi/4} \sec x\,dx$.
   - Integrate: $L = [\ln|\sec x + \tan x|]_0^{\pi/4}$.
   - Evaluate: $\ln|\sec(\pi/4)+\tan(\pi/4)| - \ln|\sec(0)+\tan(0)| = \ln|\sqrt{2}+1| - \ln|1+0| = \ln(1+\sqrt{2})$.

   Answer: \ln(1 + \sqrt2)