

Which volume method? Technique clinic

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Practice problems

1. Find the volume of the solid formed by rotating the region bounded by $y = x^3$ and $y = x$ in the first quadrant around the x-axis.
2. Find the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ around the y-axis.
3. A region is bounded by $y = x^2$ and $y = 2x$. Rotate this region around the line $x = -1$. Which method is more efficient, and what is the volume?

Solutions

1. Find the volume of the solid formed by rotating the region bounded by $y = x^3$ and $y = x$ in the first quadrant around the x-axis.

- Find intersection points: $x^3 = x \implies x(x^2 - 1) = 0$, so $x = 0$ and $x = 1$ in the first quadrant.
- Using the washer method (integrate dx): Outer radius $R(x) = x$, inner radius $r(x) = x^3$.
- Set up integral: $V = \int_0^1 \pi(x^2 - (x^3)^2) dx = \pi \int_0^1 (x^2 - x^6) dx$.
- Integrate: $\pi[\frac{1}{3}x^3 - \frac{1}{7}x^7]_0^1 = \pi(\frac{1}{3} - \frac{1}{7}) = \pi(\frac{7-3}{21}) = \frac{4\pi}{21}$.

Answer: $\frac{4\pi}{21}$

2. Find the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ around the y-axis.

- Identify boundaries: x goes from 0 to 4. y goes from 0 to 2.
- Using shell method (integrate dx): Radius is x , height is $\sqrt{x}$.
- Set up integral: $V = \int_0^4 2\pi x \sqrt{x} dx = 2\pi \int_0^4 x^{3/2} dx$.
- Integrate: $2\pi[\frac{2}{5}x^{5/2}]_0^4 = 2\pi(\frac{2}{5} \cdot 32) = \frac{128\pi}{5}$.

Answer: $\frac{128\pi}{5}$

3. A region is bounded by $y = x^2$ and $y = 2x$. Rotate this region around the line $x = -1$. Which method is more efficient, and what is the volume?

- Intersection points: $x^2 = 2x \implies x = 0, x = 2$.
- Shell method (integrate dx): Radius is $x - (-1) = x + 1$. Height is $2x - x^2$.
- Set up integral: $V = \int_0^2 2\pi(x + 1)(2x - x^2) dx$.
- Expand: $(x + 1)(2x - x^2) = 2x^2 - x^3 + 2x - x^2 = -x^3 + x^2 + 2x$.
- Integrate: $2\pi[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2]_0^2 = 2\pi(-4 + \frac{8}{3} + 4) = 2\pi(\frac{8}{3}) = \frac{16\pi}{3}$.

Answer: $\frac{16\pi}{3}$