

# Volumes by cylindrical shells

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## Practice problems

1. Find the volume of the solid generated by rotating the region bounded by  $y = x^3$ ,  $y = 0$ , and  $x = 1$  around the  $y$ -axis using the shell method.
2. Find the volume of the solid generated by rotating the region bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 4$  around the  $y$ -axis using the shell method.
3. Find the volume of the solid generated by rotating the region bounded by  $x = y^2$  and  $x = 2y$  around the  $y$ -axis using the shell method.

## Solutions

1. Find the volume of the solid generated by rotating the region bounded by  $y = x^3$ ,  $y = 0$ , and  $x = 1$  around the  $y$ -axis using the shell method.

- Identify the radius as  $x$  and the height as  $f(x) = x^3$ .
- Set up the integral:  $V = \int_0^1 2\pi x(x^3) dx$ .
- Simplify the integrand:  $V = 2\pi \int_0^1 x^4 dx$ .
- Integrate:  $V = 2\pi[\frac{1}{5}x^5]_0^1$ .
- Evaluate:  $V = 2\pi(\frac{1}{5} - 0) = \frac{2\pi}{5}$ .

**Answer:**  $\frac{2\pi}{5}$

2. Find the volume of the solid generated by rotating the region bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 4$  around the  $y$ -axis using the shell method.

- Identify the radius as  $x$  and the height as  $f(x) = \sqrt{x}$ .
- Set up the integral:  $V = \int_0^4 2\pi x(x^{1/2}) dx$ .
- Simplify the integrand:  $V = 2\pi \int_0^4 x^{3/2} dx$ .
- Integrate:  $V = 2\pi[\frac{2}{5}x^{5/2}]_0^4$ .
- Evaluate:  $V = 2\pi(\frac{2}{5} \cdot 32) = 2\pi(\frac{64}{5}) = \frac{128\pi}{5}$ .

**Answer:**  $\frac{128\pi}{5}$

3. Find the volume of the solid generated by rotating the region bounded by  $x = y^2$  and  $x = 2y$  around the  $y$ -axis using the shell method.

- Find intersection points:  $y^2 = 2y \implies y(y - 2) = 0$ , so  $y = 0$  and  $y = 2$ .
- The intersection points in  $x$  are  $x = 0$  and  $x = 4$ .
- For rotation around the  $y$ -axis, shells use  $dx$ . The height is the difference between the upper curve  $y = \sqrt{x}$  and the lower curve  $y = x/2$ .
- Set up the integral:  $V = \int_0^4 2\pi x(\sqrt{x} - \frac{x}{2}) dx$ .
- Simplify:  $V = 2\pi \int_0^4 (x^{3/2} - \frac{1}{2}x^2) dx$ .
- Integrate:  $V = 2\pi[\frac{2}{5}x^{5/2} - \frac{1}{6}x^3]_0^4$ .
- Evaluate:  $V = 2\pi(\frac{64}{5} - \frac{64}{6}) = 2\pi(\frac{384-320}{30}) = 2\pi(\frac{64}{30}) = \frac{64\pi}{15}$ .

**Answer:**  $\frac{64\pi}{15}$