

Volumes by slicing (discs and washers)

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Practice problems

1. Find the volume of the solid generated by rotating the region bounded by $y = x^3$, the x -axis, and $x = 2$ around the x -axis.
2. Find the volume of the solid formed by rotating the region bounded by $y = x^2$ and $y = 4$ around the y -axis.
3. Find the volume of the solid formed by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ around the x -axis.

Solutions

1. Find the volume of the solid generated by rotating the region bounded by $y = x^3$, the x -axis, and $x = 2$ around the x -axis.

- Identify the radius: $R(x) = x^3$.
- Set up the integral: $V = \int_0^2 \pi(x^3)^2 dx = \pi \int_0^2 x^6 dx$.
- Integrate: $\pi[\frac{1}{7}x^7]_0^2$.
- Evaluate: $\pi(\frac{128}{7} - 0) = \frac{128\pi}{7}$.

Answer: $\frac{128\pi}{7}$

2. Find the volume of the solid formed by rotating the region bounded by $y = x^2$ and $y = 4$ around the y -axis.

- Solve for x in terms of y : $x = \sqrt{y}$.
- Identify the radius: $R(y) = \sqrt{y}$.
- Set up the integral from $y = 0$ to $y = 4$: $V = \int_0^4 \pi(\sqrt{y})^2 dy = \pi \int_0^4 y dy$.
- Integrate and evaluate: $\pi[\frac{1}{2}y^2]_0^4 = \pi(8 - 0) = 8\pi$.

Answer: 8π

3. Find the volume of the solid formed by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ around the x -axis.

- Find intersection points: $x = \sqrt{x} \implies x^2 = x \implies x(x - 1) = 0$, so $x = 0$ and $x = 1$.
- Identify radii: Outer $R(x) = \sqrt{x}$, Inner $r(x) = x$.
- Set up the washer integral: $V = \pi \int_0^1 ((\sqrt{x})^2 - x^2) dx = \pi \int_0^1 (x - x^2) dx$.
- Integrate: $\pi[\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^1$.
- Evaluate: $\pi(\frac{1}{2} - \frac{1}{3}) = \pi(\frac{3-2}{6}) = \frac{\pi}{6}$.

Answer: $\frac{\pi}{6}$