Area between curves

Practice problems

1. Find the area of the region enclosed by the curve $y = x^2 - 4$ and the x-axis ($y = 0$).
2. Find the area of the region enclosed by $y = \sqrt{x}$ and $y = x^2$.
3. Find the area of the region enclosed by $x = y^2 - 1$ and $x = 1 - y^2$.

Solutions

1. Find the area of the region enclosed by the curve $y = x^2 - 4$ and the x-axis ($y = 0$).

   - Find intersections: $x^2 - 4 = 0 \implies x = \pm 2$.
   - Determine top/bottom: For $x = 0$, $y = -4$, so the x-axis ($y = 0$) is the top boundary.
   - Set up integral: $\int_{-2}^{2}(0 - (x^2 - 4))\,dx = \int_{-2}^{2}(4 - x^2)\,dx$.
   - Integrate: $[4x - \frac{1}{3}x^3]_{-2}^{2}$.
   - Evaluate: $(8 - \frac{8}{3}) - (-8 + \frac{8}{3}) = \frac{16}{3} - (-\frac{16}{3}) = \frac{32}{3}$.

   Answer: $\frac{32}{3}$

2. Find the area of the region enclosed by $y = \sqrt{x}$ and $y = x^2$.

   - Find intersections: $\sqrt{x} = x^2 \implies x = x^4 \implies x(x^3 - 1) = 0 \implies x = 0, 1$.
   - Determine top/bottom: For $x = 0.25$, $\sqrt{0.25} = 0.5$ and $0.25^2 = 0.0625$. So $\sqrt{x}$ is on top.
   - Set up integral: $\int_0^1 (\sqrt{x} - x^2)\,dx$.
   - Integrate: $[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3]_0^1$.
   - Evaluate: $(\frac{2}{3} - \frac{1}{3}) - 0 = \frac{1}{3}$.

   Answer: $\frac{1}{3}$

3. Find the area of the region enclosed by $x = y^2 - 1$ and $x = 1 - y^2$.

   - Find intersections: $y^2 - 1 = 1 - y^2 \implies 2y^2 = 2 \implies y^2 = 1 \implies y = \pm 1$.
   - Determine right/left: For $y = 0$, $x = -1$ for the first curve and $x = 1$ for the second. So $1 - y^2$ is on the right.
   - Set up integral: $\int_{-1}^{1}((1 - y^2) - (y^2 - 1))\,dy = \int_{-1}^{1}(2 - 2y^2)\,dy$.
   - Integrate: $[2y - \frac{2}{3}y^3]_{-1}^{1}$.
   - Evaluate: $(2 - \frac{2}{3}) - (-2 + \frac{2}{3}) = \frac{4}{3} - (-\frac{4}{3}) = \frac{8}{3}$.

   Answer: $\frac{8}{3}$