

Module 3 review + self-test

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Practice problems

1. Evaluate the indefinite integral $\int x^2 \cos x \, dx$.
2. Evaluate the definite integral $\int_0^1 \frac{1}{1+x^2} \, dx$.
3. Evaluate the indefinite integral $\int \frac{1}{x^2-5x+6} \, dx$.

Solutions

1. Evaluate the indefinite integral $\int x^2 \cos x \, dx$.

- Identify this as a product of a polynomial and a trig function, suggesting integration by parts.
- Let $u = x^2$ and $dv = \cos x \, dx$. Then $du = 2x \, dx$ and $v = \sin x$.
- Apply the formula: $\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$.
- Apply integration by parts again to $\int 2x \sin x \, dx$ with $u = 2x$ and $dv = \sin x \, dx$.
- Then $du = 2 \, dx$ and $v = -\cos x$.
- The second integral becomes $-2x \cos x - \int -2 \cos x \, dx = -2x \cos x + 2 \sin x$.
- Combine everything: $x^2 \sin x - (-2x \cos x + 2 \sin x) + C = x^2 \sin x + 2x \cos x - 2 \sin x + C$.

Answer: $x^2 \sin x + 2x \cos x - 2 \sin x + C$

2. Evaluate the definite integral $\int_0^1 \frac{1}{1+x^2} \, dx$.

- Recognize the integrand as the derivative of the arctangent function.
- The antiderivative of $\frac{1}{1+x^2}$ is $\arctan(x)$.
- Evaluate at the boundaries: $\arctan(1) - \arctan(0)$.
- Since $\tan(\pi/4) = 1$ and $\tan(0) = 0$, the values are $\pi/4$ and 0.

Answer: $\frac{\pi}{4}$

3. Evaluate the indefinite integral $\int \frac{1}{x^2-5x+6} \, dx$.

- Factor the denominator: $x^2 - 5x + 6 = (x-2)(x-3)$.
- Use partial fraction decomposition: $\frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$.
- Solve for constants: $1 = A(x-3) + B(x-2)$.
- Setting $x = 3$ gives $B = 1$. Setting $x = 2$ gives $A = -1$.
- The integral becomes $\int \frac{-1}{x-2} \, dx + \int \frac{1}{x-3} \, dx$.
- Integrate to get $-\ln|x-2| + \ln|x-3| + C = \ln\left|\frac{x-3}{x-2}\right| + C$.

Answer: $\ln\left|\frac{x-3}{x-2}\right| + C$