

Average value of a function; MVT for integrals

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Practice problems

1. Find the average value of $f(x) = 4x^3 - 2x$ on the interval $[0, 2]$.
2. Find the average value of $f(x) = \frac{1}{x}$ on the interval $[1, e]$.
3. For $f(x) = 3x^2$ on the interval $[0, 2]$, find the average value and then find the value c guaranteed by the Mean Value Theorem for Integrals.

Solutions

1. Find the average value of $f(x) = 4x^3 - 2x$ on the interval $[0, 2]$.

- Identify the interval width: $b - a = 2 - 0 = 2$.
- Set up the integral: $\int_0^2 (4x^3 - 2x) dx$.
- Find the antiderivative: $x^4 - x^2$.
- Evaluate the definite integral: $(2^4 - 2^2) - (0^4 - 0^2) = 16 - 4 = 12$.
- Divide by the width: $f_{\text{avg}} = \frac{12}{2} = 6$.

Answer: 6

2. Find the average value of $f(x) = \frac{1}{x}$ on the interval $[1, e]$.

- Identify the interval width: $e - 1$.
- Set up the integral: $\int_1^e \frac{1}{x} dx$.
- Find the antiderivative: $\ln|x|$.
- Evaluate the definite integral: $\ln(e) - \ln(1) = 1 - 0 = 1$.
- Divide by the width: $f_{\text{avg}} = \frac{1}{e-1}$.

Answer: $\frac{1}{e-1}$

3. For $f(x) = 3x^2$ on the interval $[0, 2]$, find the average value and then find the value c guaranteed by the Mean Value Theorem for Integrals.

- Interval width is $2 - 0 = 2$.
- Integral is $\int_0^2 3x^2 dx = [x^3]_0^2 = 8 - 0 = 8$.
- Average value is $f_{\text{avg}} = \frac{8}{2} = 4$.
- Set $f(c) = 4$, so $3c^2 = 4$.
- Solve for c : $c^2 = \frac{4}{3}$, so $c = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$ (since c must be in $[0, 2]$).

Answer: $f_{\text{avg}} = 4$, $c = \frac{2}{\sqrt{3}}$