

The integral of $\sec x$ — the historical puzzle

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Practice problems

1. Evaluate the indefinite integral $\int 5 \sec x \, dx$.
2. Evaluate the definite integral $\int_0^{\pi/6} \sec x \, dx$.
3. Evaluate the indefinite integral $\int \sec(3x) \, dx$.

Solutions

1. Evaluate the indefinite integral $\int 5 \sec x \, dx$.

- Use the linearity of the integral to pull the constant 5 outside: $5 \int \sec x \, dx$.
- Apply the standard formula $\int \sec x \, dx = \ln |\sec x + \tan x|$.
- Combine the results and add the constant of integration.

Answer: $5 \ln |\sec x + \tan x| + C$

2. Evaluate the definite integral $\int_0^{\pi/6} \sec x \, dx$.

- Find the antiderivative: $\ln |\sec x + \tan x|$.
- Evaluate at the upper limit $\pi/6$: $\sec(\pi/6) = 2/\sqrt{3}$ and $\tan(\pi/6) = 1/\sqrt{3}$.
- The sum is $3/\sqrt{3} = \sqrt{3}$, so the value is $\ln(\sqrt{3})$.
- Evaluate at the lower limit 0: $\sec(0) = 1$ and $\tan(0) = 0$, giving $\ln(1) = 0$.
- Subtract the lower limit from the upper limit.

Answer: $\ln(\sqrt{3})$

3. Evaluate the indefinite integral $\int \sec(3x) \, dx$.

- Use u-substitution: let $u = 3x$, then $du = 3 \, dx$, which means $dx = \frac{1}{3} du$.
- Rewrite the integral as $\frac{1}{3} \int \sec u \, du$.
- Apply the formula: $\frac{1}{3} \ln |\sec u + \tan u| + C$.
- Substitute $3x$ back in for u .

Answer: $\frac{1}{3} \ln |\sec(3x) + \tan(3x)| + C$