

Numerical integration: trapezoid and Simpson's rule

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Practice problems

1. Use the trapezoid rule with $n = 4$ to approximate the integral $\int_1^2 \ln(x) \, dx$.
2. Use Simpson's rule with $n = 4$ to approximate the integral $\int_0^1 \frac{1}{1+x^2} \, dx$.
3. Explain why Simpson's rule is exact for the integral $\int_0^2 (x^3 + 2x) \, dx$ regardless of the even number n chosen, and calculate the exact value using the Fundamental Theorem of Calculus to verify.

Solutions

1. Use the trapezoid rule with $n = 4$ to approximate the integral $\int_1^2 \ln(x) dx$.
 - Calculate $h = (2 - 1)/4 = 0.25$.
 - Find x values: 1, 1.25, 1.5, 1.75, 2.
 - Evaluate $f(x) = \ln(x)$: $\ln(1) = 0$, $\ln(1.25) \approx 0.223$, $\ln(1.5) \approx 0.405$, $\ln(1.75) \approx 0.560$, $\ln(2) \approx 0.693$.
 - Apply formula: $\frac{0.25}{2}[0 + 2(0.223 + 0.405 + 0.560) + 0.693]$.
 - Compute: $0.125[0 + 2(1.188) + 0.693] = 0.125[2.376 + 0.693] = 0.125[3.069] \approx 0.384$.

Answer: 0.384

2. Use Simpson's rule with $n = 4$ to approximate the integral $\int_0^1 \frac{1}{1+x^2} dx$.
 - Calculate $h = (1 - 0)/4 = 0.25$.
 - Find x values: 0, 0.25, 0.5, 0.75, 1.
 - Evaluate $f(x) = 1/(1 + x^2)$: $f(0) = 1$, $f(0.25) = 0.941$, $f(0.5) = 0.8$, $f(0.75) = 0.64$, $f(1) = 0.5$.
 - Apply formula: $\frac{0.25}{3}[1 + 4(0.941) + 2(0.8) + 4(0.64) + 0.5]$.
 - Compute: $\frac{0.25}{3}[1 + 3.764 + 1.6 + 2.56 + 0.5] = \frac{0.25}{3}[9.424] \approx 0.785$.

Answer: 0.785

3. Explain why Simpson's rule is exact for the integral $\int_0^2 (x^3 + 2x) dx$ regardless of the even number n chosen, and calculate the exact value using the Fundamental Theorem of Calculus to verify.
 - Simpson's rule is based on quadratic interpolation. The error term for Simpson's rule involves the fourth derivative of the function.
 - For $f(x) = x^3 + 2x$, the first derivative is $3x^2 + 2$, the second is $6x$, the third is 6, and the fourth is 0.
 - Since the fourth derivative is zero, the error is zero, making the approximation exact.
 - Using FTC: $\int_0^2 (x^3 + 2x) dx = [\frac{1}{4}x^4 + x^2]_0^2 = (\frac{16}{4} + 4) - (0) = 4 + 4 = 8$.

Answer: 8