

Mixed integration gauntlet I

Lesson 76 · Module 3 — Integral Calculus · The Calculus Course · dailymaths.uk/course

Practice problems

1. Evaluate the integral $\int x e^{x^2} dx$.
2. Evaluate the integral $\int \frac{1}{x^2-1} dx$.
3. Evaluate the integral $\int x^3 \cos(x) dx$.

Solutions

1. Evaluate the integral $\int x e^{x^2} dx$.

- Identify that x is a multiple of the derivative of the exponent x^2 .
- Let $u = x^2$, then $du = 2x dx$, which means $x dx = \frac{1}{2} du$.
- Substitute into the integral: $\int \frac{1}{2} e^u du$.
- Integrate to get $\frac{1}{2} e^u + C$.
- Substitute back $u = x^2$.

Answer: $\frac{1}{2} e^{x^2} + C$

2. Evaluate the integral $\int \frac{1}{x^2-1} dx$.

- Factor the denominator as $(x-1)(x+1)$.
- Use partial fraction decomposition: $\frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$.
- Solve for constants: $1 = A(x+1) + B(x-1)$. Setting $x = 1$ gives $A = 1/2$; setting $x = -1$ gives $B = -1/2$.
- Integrate $\int (\frac{1/2}{x-1} - \frac{1/2}{x+1}) dx$.
- Result is $\frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + C$.

Answer: $\frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + C$

3. Evaluate the integral $\int x^3 \cos(x) dx$.

- Use integration by parts with $u = x^3$ and $dv = \cos(x) dx$.
- First pass: $u = x^3, v = \sin(x)$. Integral becomes $x^3 \sin(x) - \int 3x^2 \sin(x) dx$.
- Second pass on $\int 3x^2 \sin(x) dx$: $u = 3x^2, v = -\cos(x)$. Integral becomes $-3x^2 \cos(x) + \int 6x \cos(x) dx$.
- Third pass on $\int 6x \cos(x) dx$: $u = 6x, v = \sin(x)$. Integral becomes $6x \sin(x) - \int 6 \sin(x) dx$.
- Final integration: $-\int 6 \sin(x) dx = 6 \cos(x)$.
- Combine all terms: $x^3 \sin(x) + 3x^2 \cos(x) - 6x \sin(x) - 6 \cos(x) + C$.

Answer: $(x^3 - 6x) \sin(x) + (3x^2 - 6) \cos(x) + C$