Strategy: choosing the right technique

Practice problems

1. Evaluate the integral $\int x^2 \cos x \, dx$.
2. Evaluate the integral $\int \frac{x+5}{x^2+2x-3} \, dx$.
3. Evaluate the integral $\int \frac{1}{\sqrt{9-x^2}} \, dx$.

Solutions

1. Evaluate the integral $\int x^2 \cos x \, dx$.

   - Identify the integrand as a product of an algebraic function x^2 and a trigonometric function $\cos x$.
   - Since there is no clear u-substitution (the derivative of x^2 is $2x$, not $\cos x$), use integration by parts.
   - Set $u = x^2$ and $dv = \cos x \, dx$, which gives $du = 2x \, dx$ and $v = \sin x$.
   - Apply the formula: $\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$.
   - Apply integration by parts again to $\int 2x \sin x \, dx$ with $u = 2x$ and $dv = \sin x \, dx$.
   - This gives $2x(-\cos x) - \int 2(-\cos x) \, dx = -2x \cos x + 2 \sin x$.

   Answer: x^2 \sin x + 2x \cos x - 2 \sin x + C

2. Evaluate the integral $\int \frac{x+5}{x^2+2x-3} \, dx$.

   - Check for u-substitution: the derivative of $x^2 + 2x - 3$ is $2x + 2$, which is not a multiple of $x + 5$.
   - Recognize this as a rational function. Factor the denominator: $x^2 + 2x - 3 = (x+3)(x-1)$.
   - Use partial fraction decomposition: $\frac{x+5}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$.
   - Solve for A: $A = \frac{-3+5}{-3-1} = \frac{2}{-4} = -\frac{1}{2}$.
   - Solve for B: $B = \frac{1+5}{1+3} = \frac{6}{4} = \frac{3}{2}$.
   - Integrate: $\int (\frac{-1/2}{x+3} + \frac{3/2}{x-1}) \, dx = -\frac{1}{2} \ln|x+3| + \frac{3}{2} \ln|x-1| + C$.

   Answer: \frac32\ln—x-1— - \frac12\ln—x+3— + C

3. Evaluate the integral $\int \frac{1}{\sqrt{9-x^2}} \, dx$.

   - Identify the form $\sqrt{a^2 - x^2}$ in the denominator, where $a = 3$.
   - This is a cue for trigonometric substitution. Set $x = 3 \sin \theta$.
   - Then $dx = 3 \cos \theta \, d\theta$ and $\sqrt{9 - x^2} = \sqrt{9(1 - \sin^2 \theta)} = 3 \cos \theta$.
   - Substitute into the integral: $\int \frac{3 \cos \theta}{3 \cos \theta} \, d\theta = \int 1 \, d\theta = \theta + C$.
   - Convert back to x: since $x/3 = \sin \theta$, then $\theta = \arcsin(x/3)$.

   Answer: \arcsin\left(\fracx3\right) + C