Partial fractions II — repeated and quadratic factors

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Practice problems

1. Evaluate the integral $\int \frac{1}{x^2-4x+4}\,dx$.
2. Evaluate the integral $\int \frac{1}{x^3+x}\,dx$.
3. Evaluate the integral $\int \frac{x^2}{x^2+1}\,dx$.

Solutions

1. Evaluate the integral $\int \frac{1}{x^2-4x+4}\,dx$.

   - Factor the denominator: $x^2 - 4x + 4 = (x-2)^2$.
   - Set up partial fractions: $\frac{1}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$.
   - Solve for constants: $1 = A(x-2) + B$. Setting $x = 2$ gives $B = 1$. Comparing coefficients of x gives $A = 0$.
   - Integrate: $\int \frac{1}{(x-2)^2}\,dx = -(x-2)^{-1} + C$.

   Answer: -\frac1x-2 + C

2. Evaluate the integral $\int \frac{1}{x^3+x}\,dx$.

   - Factor the denominator: $x(x^2+1)$.
   - Set up partial fractions: $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$.
   - Solve for constants: $1 = A(x^2+1) + (Bx+C)x$.
   - Comparing coefficients: x^2 terms: $A + B = 0$; x terms: $C = 0$; constants: $A = 1$. Thus $B = -1$.
   - Integrate: $\int (\frac{1}{x} - \frac{x}{x^2+1})\,dx = \ln|x| - \frac{1}{2}\ln(x^2+1) + C$.

   Answer: \ln—x— - \frac12\ln(xˆ2+1) + C

3. Evaluate the integral $\int \frac{x^2}{x^2+1}\,dx$.

   - Check degree: Numerator degree is 2, denominator degree is 2. The fraction is improper.
   - Perform long division: $\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$.
   - Integrate the resulting terms: $\int 1\,dx - \int \frac{1}{x^2+1}\,dx$.
   - Apply basic integration rules: $x - \arctan(x) + C$.

   Answer: x - \arctan(x) + C