Partial fractions I — distinct linear factors

Practice problems

1. Evaluate the integral $\int \frac{3x+1}{x^2-x-6}\,dx$.
2. Evaluate the integral $\int \frac{1}{x^2-4x}\,dx$.
3. Evaluate the integral $\int \frac{x^2+1}{x^3-x}\,dx$.

Solutions

1. Evaluate the integral $\int \frac{3x+1}{x^2-x-6}\,dx$.

   - Factor the denominator: $x^2 - x - 6 = (x-3)(x+2)$.
   - Set up decomposition: $\frac{3x+1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$.
   - Multiply by denominator: $3x + 1 = A(x+2) + B(x-3)$.
   - Find A by letting $x = 3$: $3(3) + 1 = A(3+2) \implies 10 = 5A \implies A = 2$.
   - Find B by letting $x = -2$: $3(-2) + 1 = B(-2-3) \implies -5 = -5B \implies B = 1$.
   - Integrate: $\int (\frac{2}{x-3} + \frac{1}{x+2})\,dx = 2\ln|x-3| + \ln|x+2| + C$.

   Answer: 2\ln—x-3— + \ln—x+2— + C

2. Evaluate the integral $\int \frac{1}{x^2-4x}\,dx$.

   - Factor the denominator: $x^2 - 4x = x(x-4)$.
   - Set up decomposition: $\frac{1}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$.
   - Multiply by denominator: $1 = A(x-4) + Bx$.
   - Find A by letting $x = 0$: $1 = A(-4) \implies A = -1/4$.
   - Find B by letting $x = 4$: $1 = B(4) \implies B = 1/4$.
   - Integrate: $\int (\frac{-1/4}{x} + \frac{1/4}{x-4})\,dx = \frac{1}{4}(\ln|x-4| - \ln|x|) + C$.

   Answer: \frac14\ln—\fracx-4x— + C

3. Evaluate the integral $\int \frac{x^2+1}{x^3-x}\,dx$.

   - Factor the denominator: $x^3 - x = x(x-1)(x+1)$.
   - Set up decomposition: $\frac{x^2+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$.
   - Multiply by denominator: $x^2 + 1 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$.
   - Find A by letting $x = 0$: $1 = A(-1)(1) \implies A = -1$.
   - Find B by letting $x = 1$: $1^2 + 1 = B(1)(2) \implies 2 = 2B \implies B = 1$.
   - Find C by letting $x = -1$: $(-1)^2 + 1 = C(-1)(-2) \implies 2 = 2C \implies C = 1$.
   - Integrate: $\int (\frac{-1}{x} + \frac{1}{x-1} + \frac{1}{x+1})\,dx = -\ln|x| + \ln|x-1| + \ln|x+1| + C$.

   Answer: \ln—\fracxˆ2-1x— + C