

Integration by parts I — the formula and LIATE

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Practice problems

1. Evaluate the integral $\int x \ln x \, dx$.
2. Evaluate the integral $\int x e^{2x} \, dx$.
3. Evaluate the integral $\int x^2 \cos x \, dx$.

Solutions

1. Evaluate the integral $\int x \ln x \, dx$.

- Using LIATE, Logarithmic comes before Algebraic, so let $u = \ln x$ and $dv = x \, dx$.
- Compute $du = \frac{1}{x} \, dx$ and $v = \frac{1}{2}x^2$.
- Apply the formula: $\int u \, dv = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} \, dx$.
- Simplify the integral: $\int \frac{1}{2}x \, dx = \frac{1}{4}x^2$.
- Combine results: $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$.

Answer: $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

2. Evaluate the integral $\int x e^{2x} \, dx$.

- Let $u = x$ and $dv = e^{2x} \, dx$.
- Compute $du = dx$ and $v = \frac{1}{2}e^{2x}$.
- Apply the formula: $\frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} \, dx$.
- Integrate the remaining part: $\int \frac{1}{2}e^{2x} \, dx = \frac{1}{4}e^{2x}$.
- Combine results: $\frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} + C$.

Answer: $\frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} + C$

3. Evaluate the integral $\int x^2 \cos x \, dx$.

- Let $u = x^2$ and $dv = \cos x \, dx$, so $du = 2x \, dx$ and $v = \sin x$.
- First pass: $x^2 \sin x - \int 2x \sin x \, dx$.
- For the second integral, let $u = 2x$ and $dv = \sin x \, dx$, so $du = 2 \, dx$ and $v = -\cos x$.
- Second pass: $\int 2x \sin x \, dx = -2x \cos x - \int -2 \cos x \, dx = -2x \cos x + 2 \sin x$.
- Combine everything: $x^2 \sin x - (-2x \cos x + 2 \sin x) + C = x^2 \sin x + 2x \cos x - 2 \sin x + C$.

Answer: $x^2 \sin x + 2x \cos x - 2 \sin x + C$