Integrals of trig functions

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Practice problems

1. Find the indefinite integral $\int \sec^3 x \tan x \, dx$.
2. Evaluate $\int \sin^5 x \, dx$.
3. Find the indefinite integral $\int \cos^3 x \sin^3 x \, dx$.

Solutions

1. Find the indefinite integral $\int \sec^3 x \tan x \, dx$.

   - Rewrite the integrand as $\sec^2 x(\sec x \tan x)$.
   - Let $u = \sec x$, then $du = \sec x \tan x \, dx$.
   - The integral becomes $\int u^2 \, du$.
   - Integrate to get $\frac{1}{3}u^3 + C$.
   - Substitute back $u = \sec x$.

   Answer: \frac13\sec^3 x + C

2. Evaluate $\int \sin^5 x \, dx$.

   - Peel off one sine: $\int (\sin^2 x)^2 \sin x \, dx$.
   - Use $\sin^2 x = 1 - \cos^2 x$ to get $\int (1 - \cos^2 x)^2 \sin x \, dx$.
   - Let $u = \cos x$, then $du = -\sin x \, dx$.
   - The integral becomes $-\int (1 - u^2)^2 \, du = -\int (1 - 2u^2 + u^4) \, du$.
   - Integrate: $-(u - \frac{2}{3}u^3 + \frac{1}{5}u^5) + C$.
   - Substitute back $u = \cos x$.

   Answer: -\cos x + \frac23\cos^3 x - \frac15\cos^5 x + C

3. Find the indefinite integral $\int \cos^3 x \sin^3 x \, dx$.

   - Both powers are odd, so we can choose either. Let's peel one cosine: $\int \cos^2 x \sin^3 x \cos x \, dx$.
   - Replace $\cos^2 x$ with $1 - \sin^2 x$: $\int (1 - \sin^2 x) \sin^3 x \cos x \, dx$.
   - Let $u = \sin x$, then $du = \cos x \, dx$.
   - The integral becomes $\int (1 - u^2)u^3 \, du = \int (u^3 - u^5) \, du$.
   - Integrate: $\frac{1}{4}u^4 - \frac{1}{6}u^6 + C$.
   - Substitute back $u = \sin x$.

   Answer: \frac14\sin^4 x - \frac16\sin^6 x + C