

u-substitution II — definite integrals and trickier cases

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Practice problems

1. Evaluate the definite integral $\int_0^{\pi/2} \sin x \cos^2 x \, dx$.
2. Find the indefinite integral $\int e^{5x-2} \, dx$.
3. Evaluate $\int_0^1 x\sqrt{x+1} \, dx$ using the substitution $u = \sqrt{x+1}$.

Solutions

1. Evaluate the definite integral $\int_0^{\pi/2} \sin x \cos^2 x \, dx$.

- Let $u = \cos x$. Then $du = -\sin x \, dx$, which means $\sin x \, dx = -du$.
- Change the limits: when $x = 0$, $u = \cos(0) = 1$. When $x = \pi/2$, $u = \cos(\pi/2) = 0$.
- The integral becomes $\int_1^0 u^2(-du) = \int_0^1 u^2 \, du$.
- Integrate u^2 to get $\frac{1}{3}u^3$ and evaluate from 0 to 1.
- $\frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 = \frac{1}{3}$.

Answer: $1/3$

2. Find the indefinite integral $\int e^{5x-2} \, dx$.

- Let $u = 5x - 2$. Then $du = 5 \, dx$, which means $dx = \frac{1}{5}du$.
- Substitute into the integral: $\int e^u \cdot \frac{1}{5}du$.
- Pull out the constant: $\frac{1}{5} \int e^u \, du$.
- Integrate e^u to get $\frac{1}{5}e^u + C$.
- Back-substitute $u = 5x - 2$ to get $\frac{1}{5}e^{5x-2} + C$.

Answer: $\frac{1}{5}e^{5x-2} + C$

3. Evaluate $\int_0^1 x\sqrt{x+1} \, dx$ using the substitution $u = \sqrt{x+1}$.

- Let $u = \sqrt{x+1}$. Then $u^2 = x+1$, so $x = u^2 - 1$.
- Differentiate $u^2 = x+1$ to get $2u \, du = dx$.
- Change limits: when $x = 0$, $u = \sqrt{1} = 1$. When $x = 1$, $u = \sqrt{2}$.
- Substitute everything: $\int_1^{\sqrt{2}} (u^2 - 1)u(2u \, du) = \int_1^{\sqrt{2}} (2u^4 - 2u^2) \, du$.
- Integrate: $[\frac{2}{5}u^5 - \frac{2}{3}u^3]_1^{\sqrt{2}}$.
- Evaluate: $(\frac{2}{5}(\sqrt{2})^5 - \frac{2}{3}(\sqrt{2})^3) - (\frac{2}{5} - \frac{2}{3}) = (\frac{8\sqrt{2}}{5} - \frac{4\sqrt{2}}{3}) - (\frac{6-10}{15}) = \frac{24\sqrt{2}-20\sqrt{2}}{15} + \frac{4}{15} = \frac{4\sqrt{2}+4}{15}$.

Answer: $\frac{4\sqrt{2}+4}{15}$