

u-substitution I — intuition and mechanics

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Practice problems

1. Evaluate the indefinite integral $\int x^2 e^{x^3} dx$.
2. Evaluate the indefinite integral $\int \sin(4x - 2) dx$.
3. Evaluate the indefinite integral $\int \frac{e^x}{1+e^x} dx$.

Solutions

1. Evaluate the indefinite integral $\int x^2 e^{x^3} dx$.

- Let $u = x^3$. Then $\frac{du}{dx} = 3x^2$, which means $dx = \frac{du}{3x^2}$.
- Substitute into the integral: $\int x^2 e^u \frac{du}{3x^2} = \frac{1}{3} \int e^u du$.
- Using the known integral of e^u , we get $\frac{1}{3}e^u + C$.
- Back-substitute $u = x^3$ to get $\frac{1}{3}e^{x^3} + C$.

Answer: $\frac{1}{3}e^{x^3} + C$

2. Evaluate the indefinite integral $\int \sin(4x - 2) dx$.

- Let $u = 4x - 2$. Then $du = 4 dx$, so $dx = \frac{1}{4}du$.
- Substitute into the integral: $\int \sin(u) \frac{1}{4}du = \frac{1}{4} \int \sin(u) du$.
- The integral of $\sin(u)$ is $-\cos(u)$, so we have $-\frac{1}{4}\cos(u) + C$.
- Back-substitute $u = 4x - 2$ to get $-\frac{1}{4}\cos(4x - 2) + C$.

Answer: $-\frac{1}{4}\cos(4x - 2) + C$

3. Evaluate the indefinite integral $\int \frac{e^x}{1+e^x} dx$.

- Let $u = 1 + e^x$. Then $du = e^x dx$.
- The numerator $e^x dx$ is exactly du .
- The integral becomes $\int \frac{1}{u} du = \ln|u| + C$.
- Back-substitute $u = 1 + e^x$ to get $\ln(1 + e^x) + C$ (absolute value is not needed as $1 + e^x$ is always positive).

Answer: $\ln(1 + e^x) + C$