

The Fundamental Theorem of Calculus, part 2 — proof and meaning

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Practice problems

1. Evaluate the definite integral $\int_0^2 (x^3 + 2x) dx$.
2. Evaluate the definite integral $\int_1^2 \frac{1}{x^2} dx$.
3. Evaluate the definite integral $\int_0^\pi \sin(x) dx$.

Solutions

1. Evaluate the definite integral $\int_0^2 (x^3 + 2x) dx$.

- Find the antiderivative of $x^3 + 2x$ using the reverse power rule: $F(x) = \frac{1}{4}x^4 + x^2$.
- Evaluate F at the upper limit $x = 2$: $F(2) = \frac{1}{4}(2)^4 + (2)^2 = \frac{16}{4} + 4 = 4 + 4 = 8$.
- Evaluate F at the lower limit $x = 0$: $F(0) = \frac{1}{4}(0)^4 + (0)^2 = 0$.
- Subtract the lower limit from the upper limit: $8 - 0 = 8$.

Answer: 8

2. Evaluate the definite integral $\int_1^2 \frac{1}{x^2} dx$.

- Rewrite the function as x^{-2} to use the power rule.
- Find the antiderivative: $F(x) = \frac{x^{-1}}{-1} = -\frac{1}{x}$.
- Evaluate at the upper limit $x = 2$: $F(2) = -\frac{1}{2}$.
- Evaluate at the lower limit $x = 1$: $F(1) = -\frac{1}{1} = -1$.
- Subtract: $-\frac{1}{2} - (-1) = -\frac{1}{2} + 1 = \frac{1}{2}$.

Answer: 1/2

3. Evaluate the definite integral $\int_0^\pi \sin(x) dx$.

- Find the antiderivative of $\sin(x)$. Since the derivative of $\cos(x)$ is $-\sin(x)$, the antiderivative is $F(x) = -\cos(x)$.
- Evaluate at the upper limit $x = \pi$: $F(\pi) = -\cos(\pi) = -(-1) = 1$.
- Evaluate at the lower limit $x = 0$: $F(0) = -\cos(0) = -(1) = -1$.
- Subtract: $1 - (-1) = 2$.

Answer: 2