

The Fundamental Theorem of Calculus, part 1 — the bridge

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Practice problems

1. Find the derivative of the function $A(x) = \int_2^x (t^3 + 5t) dt$.
2. Compute the derivative of $G(x) = \int_0^{x^4} \sin(t) dt$.
3. Find the derivative of $H(x) = \int_x^{10} e^{t^2} dt$.

Solutions

1. Find the derivative of the function $A(x) = \int_2^x (t^3 + 5t) dt$.

- Identify the integrand as $f(t) = t^3 + 5t$.
- The upper limit of integration is simply x .
- Apply the Fundamental Theorem of Calculus part 1: $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.
- Substitute x for t in the integrand.

Answer: $x^3 + 5x$

2. Compute the derivative of $G(x) = \int_0^{x^4} \sin(t) dt$.

- Identify the integrand as $f(t) = \sin(t)$ and the upper limit as $g(x) = x^4$.
- Apply the chain rule version of the theorem: $f(g(x)) \cdot g'(x)$.
- The value of the integrand at the upper limit is $\sin(x^4)$.
- The derivative of the upper limit x^4 is $4x^3$.
- Multiply the two results together.

Answer: $4x^3 \sin(x^4)$

3. Find the derivative of $H(x) = \int_x^{10} e^{t^2} dt$.

- Notice that the variable x is in the lower limit of integration.
- Use the property $\int_x^a f(t) dt = -\int_a^x f(t) dt$ to move x to the upper limit.
- Rewrite the function as $H(x) = -\int_{10}^x e^{t^2} dt$.
- Apply the Fundamental Theorem of Calculus part 1 to the integral.
- The derivative is the negative of the integrand evaluated at x .

Answer: $-e^{x^2}$