

Properties of definite integrals

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Practice problems

1. Use the properties of linearity to rewrite the integral $\int_1^3 (5x^3 - 2x + 7) dx$ as a sum of three simpler integrals.
2. Given that $\int_0^4 f(x) dx = 12$ and $\int_0^2 f(x) dx = 5$, find the value of $\int_2^4 f(x) dx$.
3. If $f(x) \geq 3$ for all x in the interval $[0, 5]$, what is the minimum possible value for $\int_0^5 f(x) dx$?

Solutions

1. Use the properties of linearity to rewrite the integral $\int_1^3 (5x^3 - 2x + 7) dx$ as a sum of three simpler integrals.

- Apply the sum rule to split the integral into three parts: $\int_1^3 5x^3 dx - \int_1^3 2x dx + \int_1^3 7 dx$.
- Apply the constant multiple rule to move the coefficients outside: $5 \int_1^3 x^3 dx - 2 \int_1^3 x dx + 7 \int_1^3 1 dx$.

Answer: $5 \int_1^3 x^3 dx - 2 \int_1^3 x dx + 7 \int_1^3 1 dx$

2. Given that $\int_0^4 f(x) dx = 12$ and $\int_0^2 f(x) dx = 5$, find the value of $\int_2^4 f(x) dx$.

- Use the additivity property: $\int_0^4 f(x) dx = \int_0^2 f(x) dx + \int_2^4 f(x) dx$.
- Substitute the known values: $12 = 5 + \int_2^4 f(x) dx$.
- Subtract 5 from both sides to isolate the unknown integral: $\int_2^4 f(x) dx = 12 - 5 = 7$.

Answer: 7

3. If $f(x) \geq 3$ for all x in the interval $[0, 5]$, what is the minimum possible value for $\int_0^5 f(x) dx$?

- Use the comparison property: since $f(x) \geq 3$, then $\int_0^5 f(x) dx \geq \int_0^5 3 dx$.
- Calculate the integral of the constant function 3 over the interval $[0, 5]$.
- The area of a rectangle with height 3 and width $5 - 0 = 5$ is $3 \times 5 = 15$.

Answer: 15