

The definite integral as a limit of sums

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Practice problems

1. Express the area under the curve $f(x) = 2x - 4$ from $x = 1$ to $x = 5$ as a definite integral. Based on the graph of the function, will the result be positive, negative, or zero?
2. A function $f(x)$ is odd, meaning $f(-x) = -f(x)$. What is the value of the definite integral $\int_{-a}^a f(x) dx$ for any $a > 0$? Explain using the concept of signed area.
3. Consider the function $f(x) = |x - 3|$ on the interval $[0, 6]$. Write the definite integral for the signed area and explain why the signed area is equal to the total geometric area in this specific case.

Solutions

1. Express the area under the curve $f(x) = 2x - 4$ from $x = 1$ to $x = 5$ as a definite integral. Based on the graph of the function, will the result be positive, negative, or zero?

- The function is $f(x) = 2x - 4$.
- The interval is from $a = 1$ to $b = 5$.
- The definite integral is $\int_1^5 (2x - 4) dx$.
- The function crosses the x axis at $2x - 4 = 0$, which is $x = 2$.
- From $x = 1$ to $x = 2$, the function is negative. From $x = 2$ to $x = 5$, the function is positive.
- The positive region (from 2 to 5) is larger than the negative region (from 1 to 2), so the result will be positive.

Answer: $\int_1^5 (2x - 4) dx$; Positive

2. A function $f(x)$ is odd, meaning $f(-x) = -f(x)$. What is the value of the definite integral $\int_{-a}^a f(x) dx$ for any $a > 0$? Explain using the concept of signed area.

- An odd function has rotational symmetry about the origin.
- This means for every point (x, y) on the graph, there is a corresponding point $(-x, -y)$.
- The area between the curve and the x axis from 0 to a is exactly the same size as the area from $-a$ to 0.
- However, because the function is odd, one of these areas is above the x axis and the other is below.
- The signed area from $-a$ to 0 is the negative of the signed area from 0 to a .
- Adding them together results in zero.

Answer: 0

3. Consider the function $f(x) = |x - 3|$ on the interval $[0, 6]$. Write the definite integral for the signed area and explain why the signed area is equal to the total geometric area in this specific case.

- The definite integral is $\int_0^6 |x - 3| dx$.
- The absolute value function $|x - 3|$ is always greater than or equal to zero for all x .
- Since $f(x) \geq 0$ for the entire interval $[0, 6]$, there is no area below the x axis.
- The signed area formula is $\text{Area}_{\text{above}} - \text{Area}_{\text{below}}$.
- Since $\text{Area}_{\text{below}} = 0$, the signed area is simply the area above the x axis.
- Therefore, the signed area equals the total geometric area.

Answer: $\int_0^6 |x - 3| dx$; The function is never negative.