

The area problem; Riemann sums

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Practice problems

1. Find the right Riemann sum R_4 for $f(x) = 3x + 2$ on the interval $[0, 2]$ with $n = 4$ sub intervals.
2. Approximate the area under $f(x) = \frac{1}{x}$ on the interval $[1, 3]$ using a left Riemann sum L_2 with $n = 2$ sub intervals.
3. For $f(x) = x^3$ on $[0, 1]$ with $n = 2$, calculate both L_2 and R_2 . Which one is the overestimate and why?

Solutions

1. Find the right Riemann sum R_4 for $f(x) = 3x + 2$ on the interval $[0, 2]$ with $n = 4$ sub intervals.

- Calculate width: $\Delta x = \frac{2-0}{4} = 0.5$.
- Identify right endpoints: $x_1 = 0.5, x_2 = 1.0, x_3 = 1.5, x_4 = 2.0$.
- Evaluate function: $f(0.5) = 3.5, f(1.0) = 5, f(1.5) = 6.5, f(2.0) = 8$.
- Sum and multiply: $R_4 = 0.5 \times (3.5 + 5 + 6.5 + 8) = 0.5 \times 23 = 11.5$.

Answer: 11.5

2. Approximate the area under $f(x) = \frac{1}{x}$ on the interval $[1, 3]$ using a left Riemann sum L_2 with $n = 2$ sub intervals.

- Calculate width: $\Delta x = \frac{3-1}{2} = 1$.
- Identify left endpoints: $x_0 = 1, x_1 = 2$.
- Evaluate function: $f(1) = 1, f(2) = 0.5$.
- Sum and multiply: $L_2 = 1 \times (1 + 0.5) = 1.5$.

Answer: 1.5

3. For $f(x) = x^3$ on $[0, 1]$ with $n = 2$, calculate both L_2 and R_2 . Which one is the overestimate and why?

- Calculate width: $\Delta x = \frac{1-0}{2} = 0.5$.
- Left endpoints: 0, 0.5. $L_2 = 0.5 \times (0^3 + 0.5^3) = 0.5 \times 0.125 = 0.0625$.
- Right endpoints: 0.5, 1. $R_2 = 0.5 \times (0.5^3 + 1^3) = 0.5 \times 1.125 = 0.5625$.
- Since $f(x) = x^3$ is increasing on $[0, 1]$, the right sum R_2 is the overestimate.

Answer: $L_2 = 0.0625$, $R_2 = 0.5625$; R_2 is the overestimate because the function is increasing.