

Optimisation I — method and classics

Lesson 55 · Module 2 — Differential Calculus · The Calculus Course · dailymaths.uk/course

Practice problems

1. A rectangle has a perimeter of 120 cm. Find the dimensions that maximize its area.
2. A rectangular garden is to be fenced on three sides, with the fourth side being a wall. If there are 100 meters of fencing, what is the maximum area?
3. An open-top box is made from a 10×10 inch square of cardboard by cutting squares of side x from the corners. Find x to maximize volume.

Solutions

1. A rectangle has a perimeter of 120 cm. Find the dimensions that maximize its area.

- Let L be length and W be width. The constraint is $2L + 2W = 120$, so $L = 60 - W$.
- The objective function is $A = L \times W = (60 - W)W = 60W - W^2$.
- Find the derivative: $A'(W) = 60 - 2W$.
- Set to zero: $60 - 2W = 0 \implies W = 30$.
- Then $L = 60 - 30 = 30$.
- Check endpoints $W = 0$ and $W = 60$, both give $A = 0$. The maximum is at $W = 30, L = 30$.

Answer: $L = 30$ \text cm, $W = 30$ \text cm

2. A rectangular garden is to be fenced on three sides, with the fourth side being a wall. If there are 100 meters of fencing, what is the maximum area?

- Let W be the two sides perpendicular to the wall and L be the side parallel to the wall.
- Constraint: $L + 2W = 100 \implies L = 100 - 2W$.
- Objective: $A = W(100 - 2W) = 100W - 2W^2$.
- Derivative: $A'(W) = 100 - 4W$.
- Critical point: $100 - 4W = 0 \implies W = 25$.
- Calculate area: $A(25) = 25(100 - 50) = 25 \times 50 = 1250$.

Answer: 1250 \text square meters

3. An open-top box is made from a 10×10 inch square of cardboard by cutting squares of side x from the corners. Find x to maximize volume.

- The height is x and the base sides are $10 - 2x$.
- Objective: $V(x) = x(10 - 2x)^2 = x(100 - 40x + 4x^2) = 4x^3 - 40x^2 + 100x$.
- Derivative: $V'(x) = 12x^2 - 80x + 100$.
- Set to zero and divide by 4: $3x^2 - 20x + 25 = 0$.
- Factor: $(3x - 5)(x - 5) = 0$.
- Critical points are $x = 5/3$ and $x = 5$.
- Domain is $0 < x < 5$. At $x = 5$, $V = 0$. At $x = 5/3$, $V = (5/3)(10 - 10/3)^2 = (5/3)(20/3)^2 = 2000/27$.

Answer: $x = 5/3$ \text inches