What the MVT really says

Lesson 50 · Module 2 — Differential Calculus · The Calculus Course · dailymaths.uk/course

Practice problems

1. Find the intervals where the function $f(x) = 2x^3 - 3x^2 - 12x$ is increasing and where it is decreasing.
2. Prove that the function $f(x) = e^x + x$ is strictly increasing for all real numbers x.
3. A function f is differentiable on $(0, \infty)$. If $f'(x) = \frac{x-3}{x^2}$, find the interval where the function is decreasing.

Solutions

1. Find the intervals where the function $f(x) = 2x^3 - 3x^2 - 12x$ is increasing and where it is decreasing.

    - Find the derivative: $f'(x) = 6x^2 - 6x - 12$.
    - Factor the derivative: $f'(x) = 6(x^2 - x - 2) = 6(x - 2)(x + 1)$.
    - Identify critical points where $f'(x) = 0$: $x = 2$ and $x = -1$.
    - Test intervals: For $x < -1$, $f'(-2) = 6(-4)(-1) = 24 > 0$ (Increasing). For $-1 < x < 2$, $f'(0) = -12 < 0$ (Decreasing). For $x > 2$, $f'(3) = 6(1)(4) = 24 > 0$ (Increasing).

    Answer: Increasing on $(-\infty, -1) \cup (2, \infty)$, decreasing on $(-1, 2)$

2. Prove that the function $f(x) = e^x + x$ is strictly increasing for all real numbers x.

    - Find the derivative: $f'(x) = e^x + 1$.
    - Analyze the range of the exponential function: e^x is always greater than 0 for all x.
    - Therefore, $f'(x) = e^x + 1$ must always be greater than 1, which means $f'(x) > 0$ for all x.
    - By the Mean Value Theorem, since the derivative is always positive, the function is strictly increasing.

    Answer: Since $f'(x) = e^x + 1 > 0$ for all x, the function is strictly increasing.

3. A function f is differentiable on $(0, \infty)$. If $f'(x) = \frac{x-3}{x^2}$, find the interval where the function is decreasing.

    - Set the derivative to be less than zero: $\frac{x-3}{x^2} < 0$.
    - Since x^2 is always positive for $x > 0$, the sign of the fraction depends only on the numerator.
    - Solve $x - 3 < 0$, which gives $x < 3$.
    - Combine this with the domain $x > 0$ to get the interval $(0, 3)$.

    Answer: The function is decreasing on the interval $(0, 3)$
