Linear approximation and differentials

Practice problems

1. Use linear approximation to estimate the value of $\sqrt[3]{28}$.
2. Use differentials to approximate the change in the volume of a sphere if the radius increases from 5 cm to 5.1 cm.
3. A square has a measured side of 10 cm. If the measurement has a maximum error of 0.05 cm, use differentials to estimate the maximum error in the calculated area.

Solutions

1. Use linear approximation to estimate the value of $\sqrt[3]{28}$.

   - Let $f(x) = x^{1/3}$. We choose $a = 27$ because it is a perfect cube near 28.
   - Calculate $f(27) = 27^{1/3} = 3$.
   - Find the derivative: $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3(\sqrt[3]{x})^2}$.
   - Evaluate the derivative at $a = 27$: $f'(27) = \frac{1}{3(3^2)} = \frac{1}{27}$.
   - Apply the formula: $L(28) = f(27) + f'(27)(28 - 27) = 3 + \frac{1}{27}(1)$.
   - Simplify to get $3 + 0.037 = 3.037$.

   Answer: 3.037

2. Use differentials to approximate the change in the volume of a sphere if the radius increases from 5 cm to 5.1 cm.

   - The volume of a sphere is $V = \frac{4}{3}\pi r^3$.
   - The differential is $dV = V'(r)dr = 4\pi r^2 dr$.
   - Substitute the known values: $r = 5$ and $dr = 0.1$.
   - Calculate $dV = 4\pi(5^2)(0.1) = 4\pi(25)(0.1)$.
   - Simplify to $100\pi(0.1) = 10\pi$.

   Answer: 10\pi \text cm^3 \approx 31.4 \text cm^3

3. A square has a measured side of 10 cm. If the measurement has a maximum error of 0.05 cm, use differentials to estimate the maximum error in the calculated area.

   - The area of a square is $A = x^2$.
   - The differential is $dA = 2xdx$.
   - Substitute the values: $x = 10$ and $dx = 0.05$.
   - Calculate $dA = 2(10)(0.05)$.
   - Simplify to $20(0.05) = 1$.

   Answer: 1 \text cm^2
