Related rates II — classic problems

Lesson 45 · Module 2 — Differential Calculus · The Calculus Course · dailymaths.uk/course

Practice problems

1. A circle's radius is increasing at a constant rate of 2 cm/s. How fast is the area of the circle increasing when the radius is 5 cm?

2. A 13 ft ladder leans against a wall. The top slides down at 1 ft/s. How fast is the bottom sliding away from the wall when the top is 5 ft above the ground?

3. A conical tank (vertex down) has a height of 10 m and a top radius of 4 m. Water flows in at 2 m^3/min. How fast is the water level rising when the water is 5 m deep?

Solutions

1. A circle's radius is increasing at a constant rate of 2 cm/s. How fast is the area of the circle increasing when the radius is 5 cm?

   - The area of a circle is $A = \pi r^2$.
   - Differentiate with respect to time t: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$.
   - Substitute the given values $r = 5$ and $\frac{dr}{dt} = 2$.
   - $\frac{dA}{dt} = 2\pi(5)(2) = 20\pi$.

   Answer: 20π cm^2/s

2. A 13 ft ladder leans against a wall. The top slides down at 1 ft/s. How fast is the bottom sliding away from the wall when the top is 5 ft above the ground?

   - Equation: $x^2 + y^2 = 13^2$.
   - Differentiate: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$.
   - Find x when $y = 5$: $x^2 + 5^2 = 13^2 \implies x^2 = 144 \implies x = 12$.
   - Substitute $x = 12, y = 5, \frac{dy}{dt} = -1$: $2(12)\frac{dx}{dt} + 2(5)(-1) = 0$.
   - $24\frac{dx}{dt} - 10 = 0 \implies \frac{dx}{dt} = \frac{10}{24} = \frac{5}{12}$.

   Answer: $\frac{5}{12}$ ft/s

3. A conical tank (vertex down) has a height of 10 m and a top radius of 4 m. Water flows in at 2 m^3/min. How fast is the water level rising when the water is 5 m deep?

   - Volume $V = \frac{1}{3}\pi r^2 h$. Ratio $\frac{r}{h} = \frac{4}{10} = \frac{2}{5}$, so $r = \frac{2}{5}h$.
   - Substitute r: $V = \frac{1}{3}\pi(\frac{2}{5}h)^2 h = \frac{4}{75}\pi h^3$.
   - Differentiate: $\frac{dV}{dt} = \frac{4}{25}\pi h^2 \frac{dh}{dt}$.
   - Substitute $\frac{dV}{dt} = 2$ and $h = 5$: $2 = \frac{4}{25}\pi(5)^2\frac{dh}{dt}$.
   - $2 = 4\pi\frac{dh}{dt} \implies \frac{dh}{dt} = \frac{1}{2\pi}$.

   Answer: $\frac{1}{2\pi}$ m/min
