

Related rates I — setting up problems

Lesson 44 · Module 2 — Differential Calculus · The Calculus Course · dailymaths.uk/course

Practice problems

1. The side length s of a square is increasing at a rate of 5 cm/s. How fast is the area A increasing when the side length is 8 cm?
2. A spherical balloon is being inflated so that its radius r increases at 2 cm/s. How fast is the volume V increasing when the radius is 3 cm? (Use $V = \frac{4}{3}\pi r^3$)
3. A point moves along the x-axis and another along the y-axis. The x-point moves away from the origin at 3 units/s, and the y-point moves toward the origin at 4 units/s. How fast is the distance z between them changing when $x = 3$ and $y = 4$?

Solutions

1. The side length s of a square is increasing at a rate of 5 cm/s. How fast is the area A increasing when the side length is 8 cm?

- The relation is $A = s^2$.
- Differentiate with respect to t : $\frac{dA}{dt} = 2s\frac{ds}{dt}$.
- Substitute $s = 8$ and $\frac{ds}{dt} = 5$.
- $\frac{dA}{dt} = 2(8)(5) = 80$.

Answer: 80 \text{ cm}^2/\text{s}

2. A spherical balloon is being inflated so that its radius r increases at 2 cm/s. How fast is the volume V increasing when the radius is 3 cm? (Use $V = \frac{4}{3}\pi r^3$)

- The relation is $V = \frac{4}{3}\pi r^3$.
- Differentiate with respect to t : $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.
- Substitute $r = 3$ and $\frac{dr}{dt} = 2$.
- $\frac{dV}{dt} = 4\pi(3^2)(2) = 4\pi(9)(2) = 72\pi$.

Answer: 72π \text{ cm}^3/\text{s}

3. A point moves along the x-axis and another along the y-axis. The x-point moves away from the origin at 3 units/s, and the y-point moves toward the origin at 4 units/s. How fast is the distance z between them changing when $x = 3$ and $y = 4$?

- The relation is $x^2 + y^2 = z^2$.
- Differentiate with respect to t : $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$.
- Find z when $x = 3, y = 4$: $z = \sqrt{3^2 + 4^2} = 5$.
- Substitute $\frac{dx}{dt} = 3$ and $\frac{dy}{dt} = -4$ (since it moves toward the origin).
- $2(3)(3) + 2(4)(-4) = 2(5)\frac{dz}{dt} \implies 18 - 32 = 10\frac{dz}{dt} \implies -14 = 10\frac{dz}{dt}$.
- $\frac{dz}{dt} = -1.4$.

Answer: -1.4 \text{ units/s}