Mixed technique clinic I

Practice problems

1. Find the derivative of $f(x) = x^3 \cos(x^2)$.
2. Find the derivative of $g(x) = \frac{\ln(x)}{x^2}$.
3. Find the derivative of $h(x) = e^{x \sin(x)}$.

Solutions

1. Find the derivative of $f(x) = x^3 \cos(x^2)$.

   - Identify the outer rule as the product rule with $u = x^3$ and $v = \cos(x^2)$.
   - Compute $u' = 3x^2$.
   - Compute v' using the chain rule: $v' = -\sin(x^2) \cdot 2x = -2x \sin(x^2)$.
   - Apply the product rule: $f'(x) = (3x^2)(\cos(x^2)) + (x^3)(-2x \sin(x^2))$.
   - Simplify the expression: $f'(x) = 3x^2 \cos(x^2) - 2x^4 \sin(x^2)$.

   Answer: f'(x) = 3x^2 \cos(x^2) - 2x^4 \sin(x^2)

2. Find the derivative of $g(x) = \frac{\ln(x)}{x^2}$.

   - Identify the outer rule as the quotient rule with $u = \ln(x)$ and $v = x^2$.
   - Compute $u' = 1/x$ and $v' = 2x$.
   - Apply the quotient rule: $g'(x) = \frac{(1/x)(x^2) - (\ln x)(2x)}{(x^2)^2}$.
   - Simplify the numerator: $(1/x)(x^2) = x$, so we have $\frac{x - 2x \ln x}{x^4}$.
   - Factor out x from the numerator and cancel with the denominator: $g'(x) = \frac{x(1 - 2 \ln x)}{x^4} = \frac{1 - 2 \ln x}{x^3}$.

   Answer: g'(x) = \frac1 - 2 \ln xx^3

3. Find the derivative of $h(x) = e^{x \sin(x)}$.

   - Identify the outer rule as the chain rule for the exponential function e^u where $u = x \sin(x)$.
   - The derivative is $h'(x) = e^{x \sin(x)} \cdot \frac{d}{dx}(x \sin(x))$.
   - To find the derivative of $x \sin(x)$, use the product rule: $(1)(\sin x) + (x)(\cos x)$.
   - Combine the results: $h'(x) = e^{x \sin(x)}(\sin x + x \cos x)$.

   Answer: h'(x) = e^x \sin(x) (\sin x + x \cos x)
