Implicit differentiation

Practice problems

1. Find $\frac{dy}{dx}$ for the relation $x^2 + 3y^2 = 11$.
2. Find $\frac{dy}{dx}$ for the relation $x^2y + y^2x = 6$.
3. Find $\frac{dy}{dx}$ for the relation $e^y + x = y$.

Solutions

1. Find $\frac{dy}{dx}$ for the relation $x^2 + 3y^2 = 11$.

   - Differentiate both sides with respect to x: $2x + 6y\frac{dy}{dx} = 0$.
   - Subtract $2x$ from both sides: $6y\frac{dy}{dx} = -2x$.
   - Divide by $6y$: $\frac{dy}{dx} = \frac{-2x}{6y}$.
   - Simplify the fraction: $\frac{dy}{dx} = -\frac{x}{3y}$.

   Answer: \fracdydx = -\fracx3y

2. Find $\frac{dy}{dx}$ for the relation $x^2y + y^2x = 6$.

   - Apply the product rule to x^2y: $(2xy + x^2\frac{dy}{dx})$.
   - Apply the product rule to y^2x: $(2yx\frac{dy}{dx} + y^2)$.
   - Combine: $2xy + x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + y^2 = 0$.
   - Group $\frac{dy}{dx}$ terms: $\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$.
   - Isolate $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{-2xy-y^2}{x^2+2xy}$.

   Answer: \fracdydx = -\frac2xy + yˆ2xˆ2 + 2xy

3. Find $\frac{dy}{dx}$ for the relation $e^y + x = y$.

   - Differentiate both sides: $e^y\frac{dy}{dx} + 1 = \frac{dy}{dx}$.
   - Move $\frac{dy}{dx}$ terms to one side: $e^y\frac{dy}{dx} - \frac{dy}{dx} = -1$.
   - Factor out $\frac{dy}{dx}$: $\frac{dy}{dx}(e^y - 1) = -1$.
   - Isolate $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{-1}{e^y-1}$.
   - Alternatively, multiply top and bottom by -1: $\frac{dy}{dx} = \frac{1}{1-e^y}$.

   Answer: \fracdydx = \frac11 - eˆy
