

Derivatives of inverse trig functions

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Practice problems

1. Find the derivative of $f(x) = \arcsin(\sqrt{x})$.
2. Find the derivative of $g(x) = \arctan(e^{2x})$.
3. Find the derivative of $h(x) = \ln(\arctan(x))$.

Solutions

1. Find the derivative of $f(x) = \arcsin(\sqrt{x})$.

- Identify the outer function as $\arcsin(u)$ and the inner function as $u = \sqrt{x}$.
- The derivative of $\arcsin(u)$ is $\frac{1}{\sqrt{1-u^2}}$.
- The derivative of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$.
- Apply the chain rule: $f'(x) = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$.
- Simplify the expression: $f'(x) = \frac{1}{2\sqrt{x}\sqrt{1-x}}$.

Answer: $f'(x) = \frac{1}{2\sqrt{x}\sqrt{1-x}}$

2. Find the derivative of $g(x) = \arctan(e^{2x})$.

- Identify the outer function as $\arctan(u)$ and the inner function as $u = e^{2x}$.
- The derivative of $\arctan(u)$ is $\frac{1}{1+u^2}$.
- The derivative of e^{2x} is $2e^{2x}$ using the chain rule.
- Apply the chain rule: $g'(x) = \frac{1}{1+(e^{2x})^2} \cdot 2e^{2x}$.
- Simplify the exponent: $(e^{2x})^2 = e^{4x}$.

Answer: $g'(x) = \frac{2e^{2x}}{1+e^{4x}}$

3. Find the derivative of $h(x) = \ln(\arctan(x))$.

- Identify the outer function as $\ln(u)$ and the inner function as $u = \arctan(x)$.
- The derivative of $\ln(u)$ is $\frac{1}{u}$.
- The derivative of $\arctan(x)$ is $\frac{1}{1+x^2}$.
- Apply the chain rule: $h'(x) = \frac{1}{\arctan(x)} \cdot \frac{1}{1+x^2}$.

Answer: $h'(x) = \frac{1}{\arctan(x)(1+x^2)}$