Derivative of ln x; logarithmic differentiation

Practice problems

1. Find the derivative of $f(x) = \ln(x^3 + 7x)$.
2. Use logarithmic differentiation to find the derivative of $y = x^5 \sin x$.
3. Find the derivative of $f(x) = x^{\cos x}$ using logarithmic differentiation.

Solutions

1. Find the derivative of $f(x) = \ln(x^3 + 7x)$.

   - Identify the inner function $u = x^3 + 7x$.
   - Compute the derivative of the inner function: $u' = 3x^2 + 7$.
   - Apply the formula $\frac{d}{dx}\ln(u) = \frac{u'}{u}$.
   - Substitute the expressions: $f'(x) = \frac{3x^2+7}{x^3+7x}$.

   Answer: $f'(x) = \frac{3x^2+7}{x^3+7x}$

2. Use logarithmic differentiation to find the derivative of $y = x^5 \sin x$.

   - Take the natural log of both sides: $\ln y = \ln(x^5 \sin x)$.
   - Use log laws to expand: $\ln y = 5\ln x + \ln(\sin x)$.
   - Differentiate implicitly: $\frac{1}{y}\frac{dy}{dx} = \frac{5}{x} + \frac{\cos x}{\sin x}$.
   - Simplify the trig term: $\frac{1}{y}\frac{dy}{dx} = \frac{5}{x} + \cot x$.
   - Multiply by y: $\frac{dy}{dx} = (x^5 \sin x)(\frac{5}{x} + \cot x)$.

   Answer: $\frac{dy}{dx} = (x^5 \sin x)(\frac{5}{x} + \cot x)$

3. Find the derivative of $f(x) = x^{\cos x}$ using logarithmic differentiation.

   - Set $y = x^{\cos x}$ and take the natural log: $\ln y = \ln(x^{\cos x})$.
   - Bring the exponent down: $\ln y = (\cos x)\ln x$.
   - Differentiate using the product rule: $\frac{1}{y}\frac{dy}{dx} = (-\sin x)\ln x + (\cos x)\frac{1}{x}$.
   - Isolate $\frac{dy}{dx}$ by multiplying by y: $\frac{dy}{dx} = x^{\cos x}(\frac{\cos x}{x} - \sin x \ln x)$.

   Answer: $f'(x) = x^{\cos x}(\frac{\cos x}{x} - \sin x \ln x)$
