

Derivative of e to the x and why e is special

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Practice problems

1. Find the derivative of $f(x) = 7^x$.
2. Find the derivative of $f(x) = e^{5x^3}$.
3. Find the derivative of $f(x) = xe^{2x}$.

Solutions

1. Find the derivative of $f(x) = 7^x$.

- Identify the base $a = 7$.
- Apply the formula $\frac{d}{dx}a^x = a^x \ln a$.
- Substitute 7 for a to get $7^x \ln 7$.

Answer: $f'(x) = 7^x \ln 7$

2. Find the derivative of $f(x) = e^{5x^3}$.

- Identify the outer function as e^u and the inner function as $u = 5x^3$.
- The derivative of the outer function is e^{5x^3} .
- The derivative of the inner function is $15x^2$.
- Multiply them together using the chain rule: $15x^2 e^{5x^3}$.

Answer: $f'(x) = 15x^2 e^{5x^3}$

3. Find the derivative of $f(x) = xe^{2x}$.

- Use the product rule with $u = x$ and $v = e^{2x}$.
- Compute $u' = 1$ and $v' = 2e^{2x}$ using the chain rule.
- Apply the product rule: $f'(x) = u'v + uv' = (1)(e^{2x}) + (x)(2e^{2x})$.
- Factor out e^{2x} to simplify: $e^{2x}(1 + 2x)$.

Answer: $f'(x) = e^{2x}(1 + 2x)$