Chain rule — technique clinic

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Practice problems

1. Find the derivative of $y = x \ln(x^2 + 1)$.
2. Find the derivative of $y = \sin^4(3x)$.
3. Find the derivative of $y = \frac{e^{2x}}{\cos(x)}$.

Solutions

1. Find the derivative of $y = x\ln(x^2+1)$.

   - Identify this as a product of $u = x$ and $v = \ln(x^2+1)$.
   - The derivative of u is $u' = 1$.
   - The derivative of v requires the chain rule: $v' = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$.
   - Apply the product rule: $y' = u'v + uv' = 1 \cdot \ln(x^2+1) + x \cdot \frac{2x}{x^2+1}$.
   - Simplify the expression to $\ln(x^2+1) + \frac{2x^2}{x^2+1}$.

   Answer: \fracdydx = \ln(xˆ2 + 1) + \frac2xˆ2xˆ2 + 1

2. Find the derivative of $y = \sin^4(3x)$.

   - Rewrite the function as $y = (\sin(3x))^4$.
   - The outermost layer is the power of 4. Its derivative is $4(\sin(3x))^3$.
   - The next layer is the sine function. Its derivative is $\cos(3x)$.
   - The innermost layer is $3x$. Its derivative is 3.
   - Multiply all layers together: $y' = 4\sin^3(3x) \cdot \cos(3x) \cdot 3$.
   - Simplify to $12\sin^3(3x)\cos(3x)$.

   Answer: \fracdydx = 12\sinˆ3(3x)\cos(3x)

3. Find the derivative of $y = \frac{e^{2x}}{\cos(x)}$.

   - Identify this as a quotient with $u = e^{2x}$ and $v = \cos(x)$.
   - Find u' using the chain rule: $u' = e^{2x} \cdot 2 = 2e^{2x}$.
   - Find v': $v' = -\sin(x)$.
   - Apply the quotient rule: $y' = \frac{u'v - uv'}{v^2} = \frac{(2e^{2x})(\cos(x)) - (e^{2x})(-\sin(x))}{(\cos(x))^2}$.
   - Factor out e^{2x} in the numerator: $y' = \frac{e^{2x}(2\cos(x) + \sin(x))}{\cos^2(x)}$.

   Answer: \fracdydx = \fraceˆ2x(2\cos(x) + \sin(x))\cosˆ2(x)
