

The chain rule — the most important rule (intuition)

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Practice problems

1. Find the derivative of $y = \ln(x^2 + 3)$.
2. Find the derivative of $y = \cos(4x^3)$.
3. Find the derivative of $y = (5x^2 - 2)^7$.

Solutions

1. Find the derivative of $y = \ln(x^2 + 3)$.

- Identify the outer function $f(u) = \ln(u)$ and the inner function $u = x^2 + 3$.
- Find the derivative of the outer function: $f'(u) = \frac{1}{u}$.
- Find the derivative of the inner function: $u' = 2x$.
- Apply the chain rule: $\frac{dy}{dx} = \frac{1}{x^2+3} \cdot 2x$.

Answer: $\frac{2x}{x^2+3}$

2. Find the derivative of $y = \cos(4x^3)$.

- Identify the outer function as cosine and the inner function as $4x^3$.
- The derivative of the outer function is $-\sin(u)$, evaluated at $u = 4x^3$.
- The derivative of the inner function $4x^3$ is $12x^2$.
- Multiply them together: $-\sin(4x^3) \cdot 12x^2$.

Answer: $-12x^2 \sin(4x^3)$

3. Find the derivative of $y = (5x^2 - 2)^7$.

- Identify the outer function as u^7 and the inner function as $u = 5x^2 - 2$.
- Differentiate the outer layer: $7u^6$, which is $7(5x^2 - 2)^6$.
- Differentiate the inner layer: $\frac{d}{dx}(5x^2 - 2) = 10x$.
- Multiply the results: $7(5x^2 - 2)^6 \cdot 10x$.

Answer: $70x(5x^2 - 2)^6$