Derivatives of sin and cos from first principles

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Practice problems

1. Find the derivative of $f(x) = 3\sin(x) - 4\cos(x)$.
2. Find the derivative of $g(x) = x^2 \cos(x)$.
3. Find the derivative of $h(x) = \frac{\cos(x)}{x+1}$.

Solutions

1. Find the derivative of $f(x) = 3\sin(x) - 4\cos(x)$.

    - Apply the linearity of the derivative to separate the terms: $f'(x) = 3\frac{d}{dx}\sin(x) - 4\frac{d}{dx}\cos(x)$.
    - Use the known derivatives: $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{dx}\cos(x) = -\sin(x)$.
    - Substitute these into the expression: $f'(x) = 3\cos(x) - 4(-\sin(x))$.
    - Simplify the signs: $f'(x) = 3\cos(x) + 4\sin(x)$.

    Answer: f’(x) = 3\cos(x) + 4\sin(x)

2. Find the derivative of $g(x) = x^2\cos(x)$.

    - Identify this as a product of $u(x) = x^2$ and $v(x) = \cos(x)$.
    - Apply the product rule: $g'(x) = u'(x)v(x) + u(x)v'(x)$.
    - Compute the individual derivatives: $u'(x) = 2x$ and $v'(x) = -\sin(x)$.
    - Substitute back: $g'(x) = (2x)\cos(x) + (x^2)(-\sin(x))$.
    - Simplify the expression: $g'(x) = 2x\cos(x) - x^2\sin(x)$.

    Answer: g’(x) = 2x\cos(x) - xˆ2\sin(x)

3. Find the derivative of $h(x) = \frac{\cos(x)}{x+1}$.

    - Identify this as a quotient of $u(x) = \cos(x)$ and $v(x) = x + 1$.
    - Apply the quotient rule: $h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$.
    - Compute the individual derivatives: $u'(x) = -\sin(x)$ and $v'(x) = 1$.
    - Substitute into the formula: $h'(x) = \frac{(-\sin(x))(x+1) - (\cos(x))(1)}{(x+1)^2}$.
    - Simplify the numerator: $h'(x) = \frac{-(x+1)\sin(x) - \cos(x)}{(x+1)^2}$.

    Answer: h’(x) = \frac-(x+1)\sin(x) - \cos(x)(x+1)ˆ2
